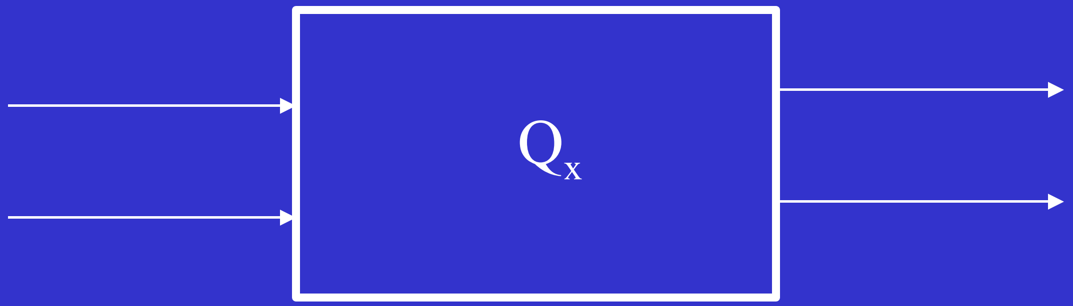
On Box Models

This is based on an old lecture that was on indoor air quality. We may return to some of the details of this topic, but for now, it is important that you learn some of the basics of box models that are used frequently in the field of atmospheric chemistry and air quality modeling. Many problems can be treated as a simple 'box' or reservoir where the amount of a particular substance X is often labeled as Q_x, and the arrows into and out of box represent fluxes of the substance X into and out of the box, respectively.

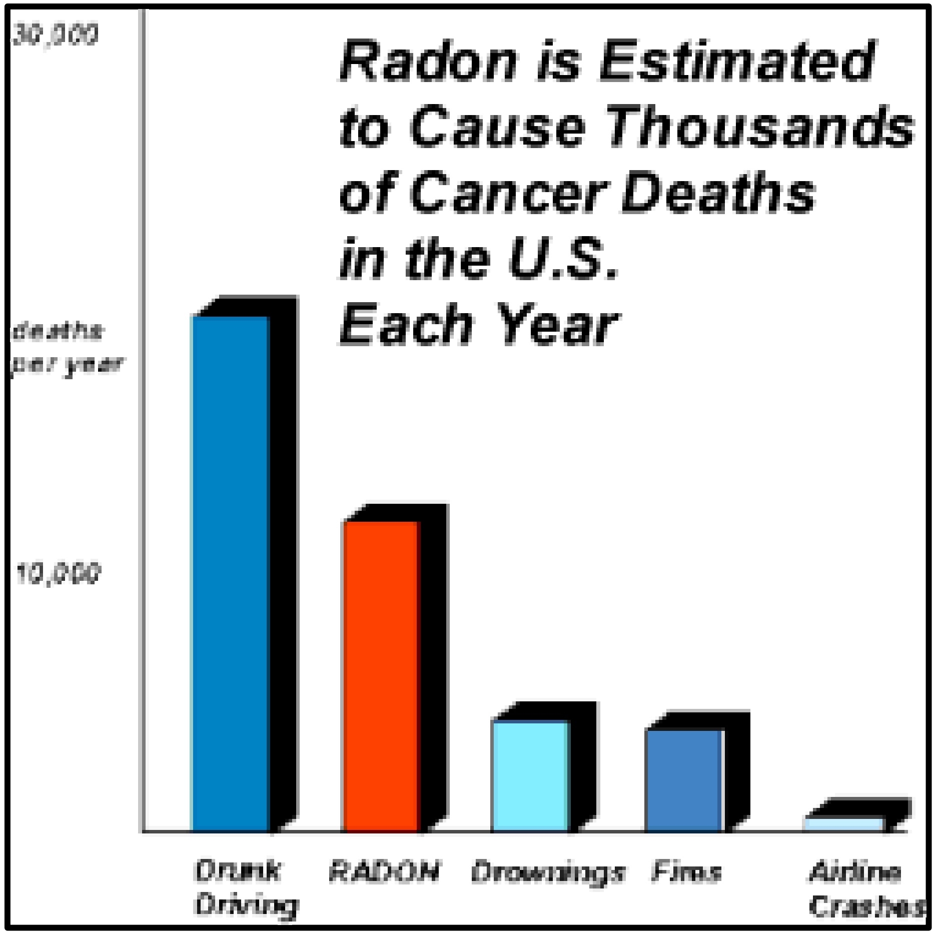


Risks from Radon (Rn) Exposure

Main health effect: lung cancer

- Among non-smokers, 10-20 cases/1000 people
- Add Rn at 1 pCi/L, 13-33 cases/1000 people
- Smokers' risk is about 15 times that of non-smokers'; due largely to the affinity of Rn and its daughters for smoke particles
- Comparison to risks from other sources

Note – a pCi/L is a unit called "picocurie per L". You can guess that it was named after Marie Curie, someone who studied radioactive elements.


Radon is Estimated to Cause Thousands of Cancer Deaths in the U.S. Each Year
30,000
deaths per year
10,000
Drunk Driving
RADON
Drownings
Fires
Airline Crashes

Estimating Exposure

Epidemiologists search for patterns in disease or illness that can reveal relationships to exposure to contaminants. One tool that modelers use to assess exposure to pollutants is the “box model”

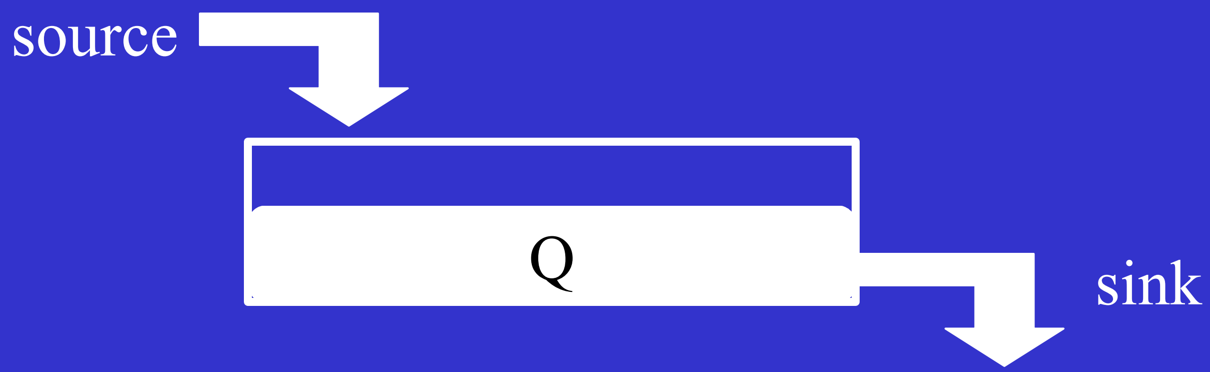
<u>The Box Model</u>
A simple tool for describing a situation or problem

Three basic elements: reservoir (box); sources; sinks

A simple example - the bathtub model:

“Source” = spigot
“Sink” = drain
“Reservoir” = tub

We often call Q the ‘burden’, since it represents a total mass of stuff, the more stuff there is, the more of a burden it is to move it around or remove it altogether.



How much water is in the tub? We express the total amount as Q

Q depends on relative sizes of source and sink; i.e., how much water is flowing from spigot and through drain. Usually, as the amount in the 'tub' or reservoir increases, the rate at which it drains (the "sink" term) increases proportionately.

What is the "residence time" of water in the tub? We express this as τ (the Greek letter "tau")

τ depends on amount of water in tub and loss rate as follows:

$$\tau = Q/\text{sink}$$

The Steady-State Concept

This allows us to set up an equation that we can solve for one of the variables…either the source, or the sink, or the amount in the reservoir.

- A simplifying assumption: Q, the amount of material in the reservoir, is constant (i.e., $dQ/dt = 0$)

- As a consequence, the amount of material entering the reservoir must equal the amount leaving; that is, the source, S (sometimes called P for ‘production rate’) must equal the sink, which is typically called L. So steady state is often referred to as the condition when $P = L$ (“P equals L”)

- Then the residence time is $\tau = Q/L$ or Q/P

Let's check our units:

$$Q/P = \tau \text{ , or } Q = P \times \tau$$

so Burden = Production x Residence Time

kg = (kg seconds^{-1}) x seconds

For air pollution, it's usually more convenient to work with the concentration of a species X, than it is to work with the mass burden Q of species X (written as Q^X)

$$Q^X / \text{Volume} = [X]$$

$$[X] \times V = P\tau$$

(concentration) x (volume) = (production) x (residence time)

(mass/volume) x volume = (mass/time) x time

$$(\mu g\ m^{-3}) \times (m^3) = (g\ s^{-1}) \times (s)$$

Recall that our goal here is to try to estimate exposure to some pollutant based on things we can measure, such as the amount of the pollutant present, how quickly air enters or leaves a room, etc.

Radon sources and sinks:

- Seepage; typically 0.01 – 10 pCi L^{-1} hr^{-1}
- Ventilation; exchange time varies from 0.5 – 4 hr
- Radioactive decay; τ_{Rn} = 3.8 day = 91 hr

Assume steady-state:

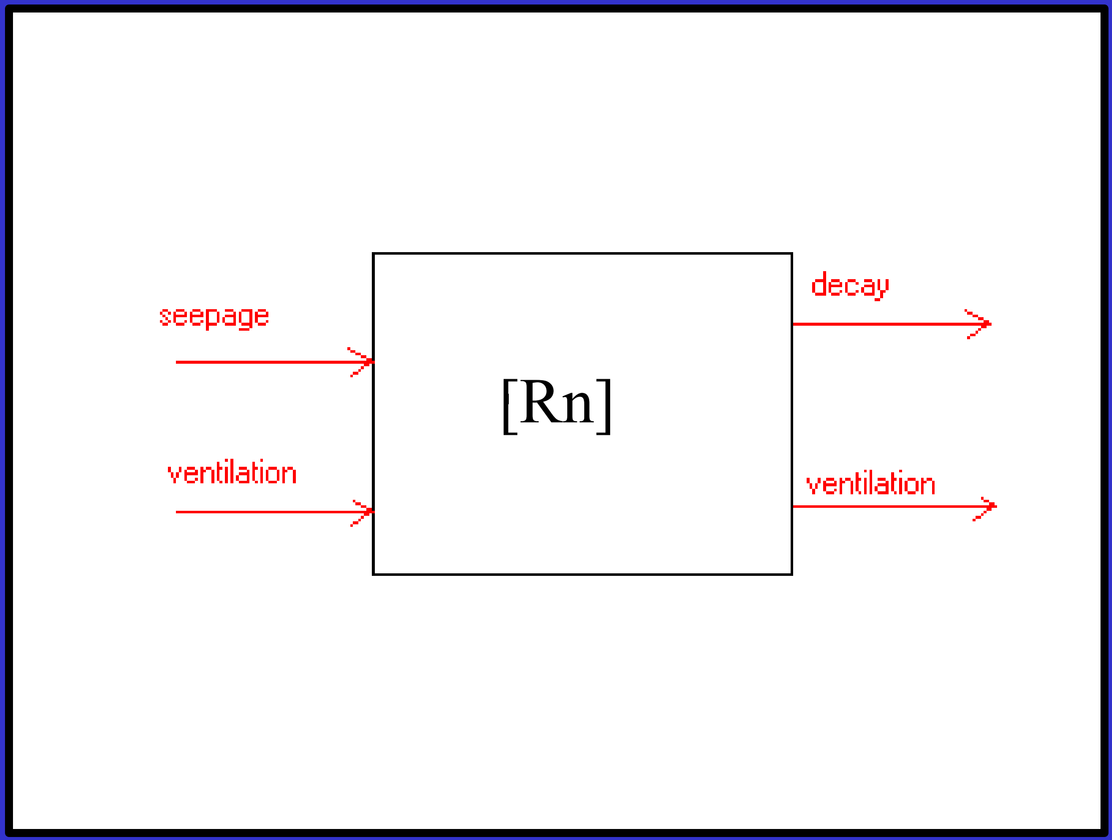
At steady-state, production equals loss, so

$$\Sigma_k P_k = \Sigma_j L_j$$

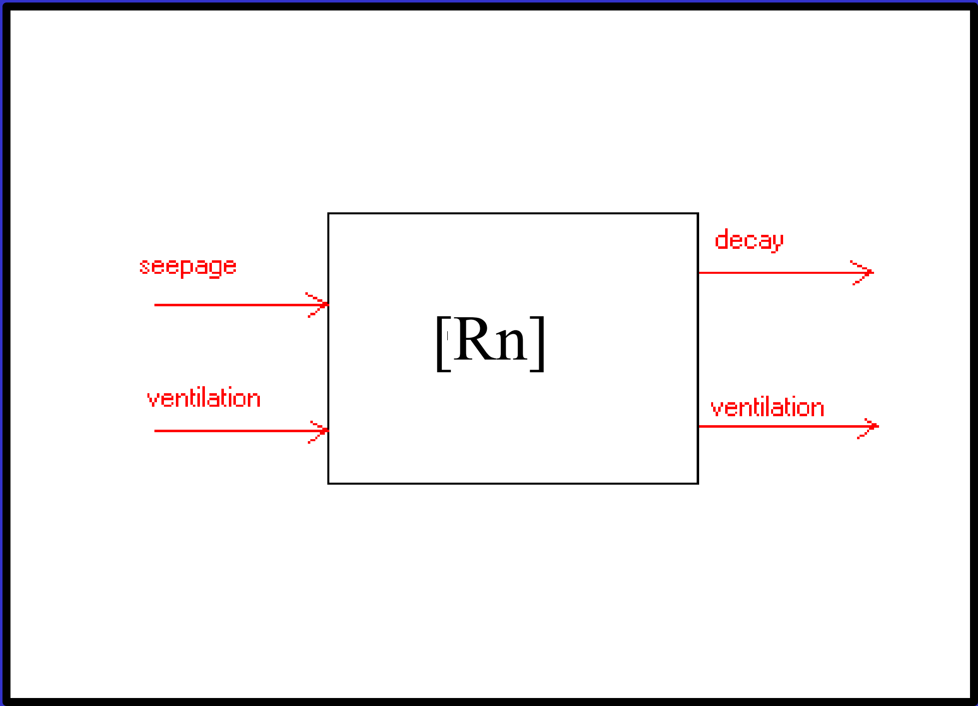
Derive steady-state concentration of indoor Rn:

Since Rn sources are given as pCi L^{-1}, we'll use the term "S" instead of "P"

Schematically, we combine these terms into a figure



The seepage source is denoted S_{Rn} and is given in units of pCi L^{-1} hr^{-1}.



The ventilation source is denoted S_v and is equivalent to the concentration of Rn in ambient air ($[Rn]_o$) divided by the ventilation time, τ_v. The units on S_v are pCi L^{-1} hr^{-1}.

The decay loss is denoted L_d, and is given by the indoor concentration of Rn ([Rn]) divided by the decay time or half-life, τ_d.

The ventilation sink is denoted L_v. Analogous to the ventilation source, $L_v = q^{Rn}/\tau_v$.

Assuming steady-state, the sum of the sources is equal to the sum of the sinks:

$$S_{Rn} + S_v = L_d + L_v$$

$$S_{Rn} + ([Rn]_o/\tau_v) = ([Rn]/\tau_d) + ([Rn]/\tau_v)$$

Multiplying through by τ_v:

$$S_{Rn}\tau_v + [Rn]_o = [Rn]\,[(\tau_v/\tau_d) + 1]$$

Rearranging:

$$[Rn] = [Rn]_o + S_{Rn}\tau_v]/\,[(\tau_v/\tau_d) + 1]$$

There are some special cases to consider that can simplify this formulation:

(1) If the ventilation time is much shorter than the decay time ($\tau_v << \tau_d$), then the denominator of the above expression is about 1. Therefore, $[Rn] = [Rn]_o + S_{Rn}\tau_v$

(2) If the ventilation time is very short, the expression in (1) can further reduce to $[Rn] = [Rn]_o$, indicating that the indoor concentrations are largely controlled by the outdoor concentrations.

(3) If the ventilation time is very long, then $[Rn] = S_{Rn}\tau_v$ and indoor concentrations are largely determined by the seepage source.

Given some average conditions:

S_{Rn} = 0.5 pCi L^{-1} hr^{-1}

τ_v = 1 hr

$[Rn]_o$ = 0.4 pCi L^{-1}

What is the indoor Rn concentration?

We can used the expression derived above: $[Rn] = [Rn]_o + S_{Rn}\tau_v$ because the 1 hour ventilation time is very much shorter than the 91 hour decay half-life of Rn. Substituting appropriate values, we obtain an indoor radon concentration of 0.9 pCi L^{-1}.

Problem: A swimming pool (volume = 3750 m^3) loses water by evaporation (10 g min^{-1}) and by splashing from playing children (100 L hr^{-1}). Water is continuously added from a hose at a rate to maintain the water level. What is that rate?

The only source in this problem is the hose, S_h. The loss processes are splashing, L_{splash}, and evaporation, L_{evap}. Since we want to maintain a constant water level (i.e., steady-state), the source must equal the sum of the losses.

$$S_h = L_{splash} + L_{evap}$$

L_{evap} is given as 10 g min^{-1}, so we must convert to L hr^{-1} for units consistency.

$$(10\ g/min) * (1\ L/1000\ g) * (60\ min/1\ hr) = 0.6\ L\ hr^{-1}$$

$$S_h = 100\ L\ hr^{-1} + 0.6\ L\ hr^{-1} = 100.6\ L\ hr^{-1}$$