Vorticity on a sphere

Spherical harmonics

(some useful properties)

$$\psi(\lambda,\mu)=\sum_n\sum_m \psi_n^m P_n^m(\mu)e^{-im\lambda}$$

$$\nabla^2\psi_n^m=-\frac{n(n+1)}{a^2}\psi_n^m$$

$$\frac{\partial\psi_n^m}{\partial\lambda}=im\psi_n^m$$

$$(1-\mu^2)\frac{\partial\psi_n^m}{\partial\mu}=-n\varepsilon_{n+1}^m\psi_{n+1}^m+(n-1)\varepsilon_n^m\psi_{n-1}^m$$

$$\varepsilon_n^m=\sqrt{\frac{n^2-m^2}{4n^2-1}}$$

Advection with spherical harmonics

$$\frac{\partial q_n^m}{\partial t} = -\vec{V} \cdot \nabla q = -\nabla \cdot \left(\vec{V} q\right)$$

Non-divergent!

$$\frac{\partial q_n^m}{\partial t} = -\alpha(F,G)_n^m$$

$$\alpha(F,G)_n^m = \frac{1}{\cos^2\phi}\left(\frac{\partial F_n^m}{\partial \lambda} + \cos\phi \frac{\partial G_n^m}{\partial \phi}\right)$$

$$F = \frac{\cos\phi}{a} uq \qquad G = \frac{\cos\phi}{a} vq$$

A model

Remember q, the potential vorticity q = ζ+f

$$\frac{\partial q}{\partial t} = -\nabla \cdot \left(\vec{V} q\right) + K\nabla^2 q - Dq$$

Resolved scale motion

Unresolved motion

Linear damping "drag"

Lab assignment

What will the weather be on Tuesday in Boulder?

Make a 5-day weather forecast from Thursday using the non-divergent barotropic vorticity equation on a sphere

$$\frac{\partial \zeta}{\partial t} = -\nabla \cdot \left(\vec{V}(\zeta + f) \right) + K\nabla^2 \zeta - D\zeta$$

Use a centered finite difference for time

Use spherical harmonics to represent the spatial structure.

Include diffusion and friction.

(extra part, include mountain topography)

Implementing model code

- What is the state variable?
  (a series of complex coefficients)

- Obtain vorticity field on a grid
- Derive spectral u and v from spectral vorticity
- Obtain u and v on a grid
- Calculate non-linear fluxes on a grid
  (*uq* and *vq*)
- Assign drag ("momentum source")
- Convert fluxes to spectral form
- Compute flux divergence in spectral form ("alpha")
  (just the advection, as per last week)
- Time step the state variable (in spectral form)

Spherical harmonic synthesis

Spherical harmonic analysis

Remember to output state every so often (say, 6 hours?)

This is just the same as last week, but now we also have u and v

Initialization?

We start with known (observed) geopotential.

Can we use this to initialize our model?

Notice from momentum equation, drag acts on relative vorticity, not absolute vorticity. How can we do this in our model? (hint, think about the alpha operator)

See code

- ~dcn/ATOC7500/week09
- Fft991.F, Spherical.F90
- Also, vorg_20060313_1800.txt
- (and topog.txt)
- Also, also, advection.f90

Spherical harmonics

- subroutine sph_init(nlon,nlat,ntrn,ltriang)
- subroutine sph_anal(nlon,nlat,nlev,grid,ntrn,spec)
- subroutine sph_synth(ntrn,nlev,spec,nlon,nlat,grid,nex_in)
- subroutine sph_anal_alpha(nlon,nlat,nlev,aunit,ag,bunit,bg,ntrn,alphas)
- subroutine sph_uvcos(ntrn, nlev, vors, divs, ucos, vcos)
- subroutine sph_lapl(ntrn,nlev,fldmn,lapmn,pow_in)