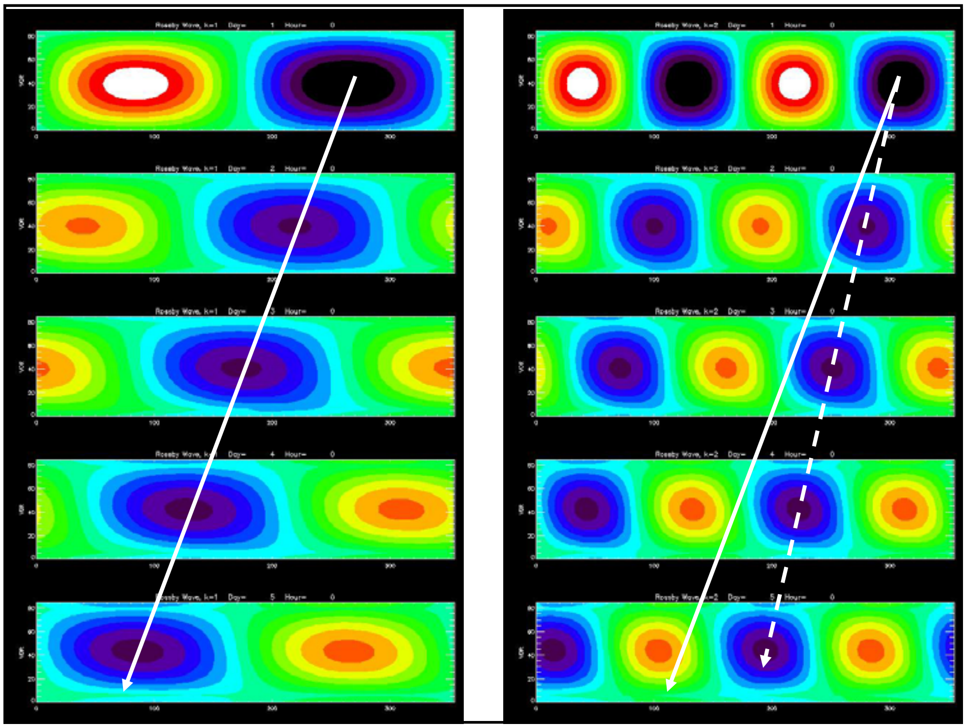
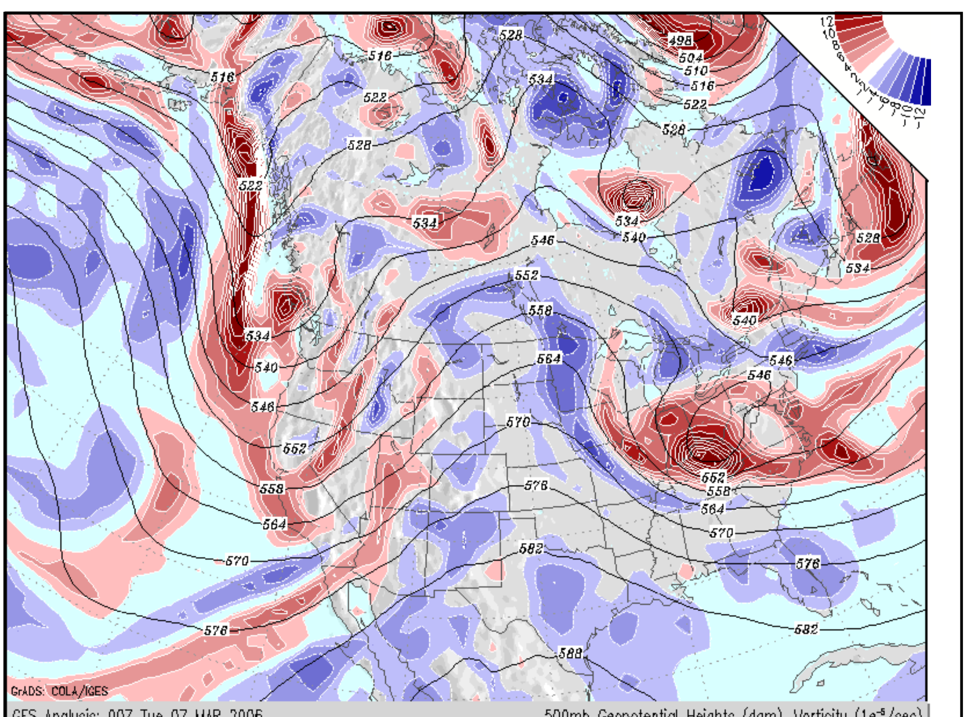
Tracer transport model
(plume dispersion)
and the spectral method

Discussion from last week

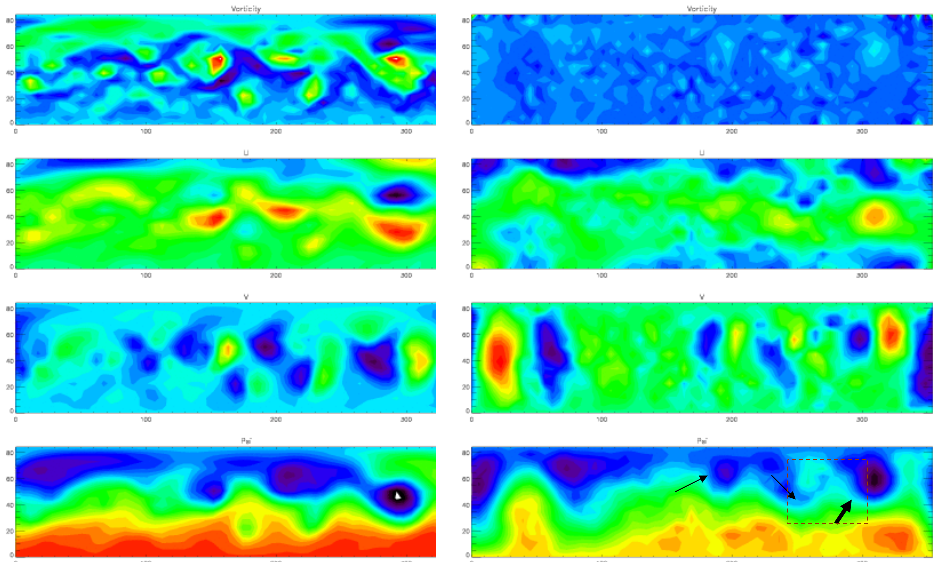
- Does your model correctly simulate Rossby waves?
- Does your model simulate all wave number just as well?
- How accurate is your prediction?
- Does your solution remains stable for long simulations? (Why?)
- How does horizontal diffusion affect results?
- How does the addition of surface drag effect results?
- ***What else can we add to the model to make it more realistic?***

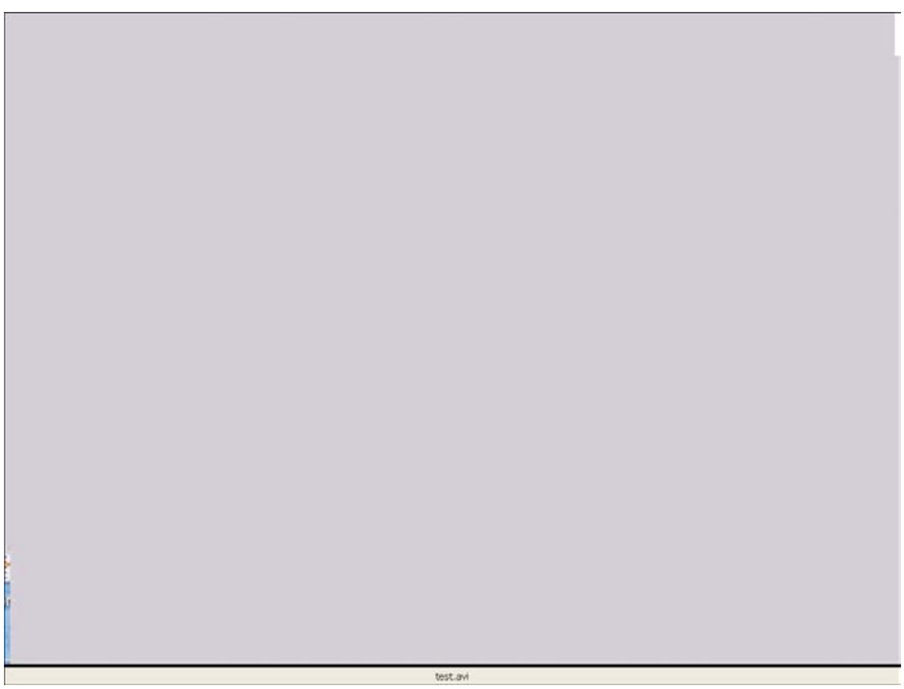



GrADS: COLA/IGES
GFS Analysis: 00Z Tue 07 MAR 2006
500mb Geopotential Heights (dam), Vorticity (1e-5/sec)

Last Thursday

+5 days (today)





Issues

- Inaccurate advection ultimately screws things up (need more accurate advection)
- Advection is ultimately dispersive
  i.e., pattern smoothes out. (need more accurate advection)
- Seems to be problems at boundaries
  (consider a sphere)

- For long term grid point models, need more accurate way of representing advection, but we know spectral models have perfect advection!

Spherical harmonics

(some useful properties)

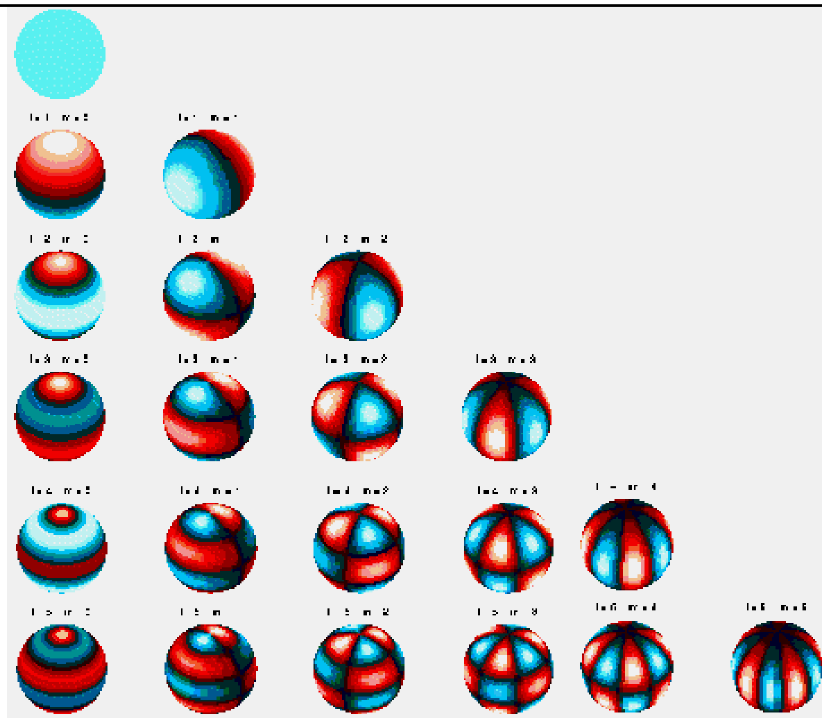
$$\psi(\lambda,\mu) = \sum_n \sum_m \psi_n^m P_n^m(\mu) e^{-im\lambda}$$

$$\nabla^2 \psi_n^m = -\frac{n(n+1)}{a^2}\psi_n^m$$

$$\frac{\partial \psi_n^m}{\partial \lambda} = im\psi_n^m$$

$$(1-\mu^2)\frac{\partial \psi_n^m}{\partial \mu} = -n\varepsilon_{n+1}^m \psi_{n+1}^m + (n-1)\varepsilon_n^m \psi_{n-1}^m$$

$$\varepsilon_n^m = \sqrt{\frac{n^2-m^2}{4n^2-1}}$$



Advection with spherical harmonics

$$\frac{\partial q_n^m}{\partial t} = -\vec{V} \cdot \nabla q = -\nabla \cdot \left(\vec{V} q\right)$$

Non-divergent!

$$\frac{\partial q_n^m}{\partial t} = -\alpha(F,G)_n^m$$

$$\alpha(F,G)_n^m = \frac{1}{\cos^2\phi}\left(\frac{\partial F_n^m}{\partial \lambda} + \cos\phi \frac{\partial G_n^m}{\partial \phi}\right)$$

$$F = \frac{\cos\phi}{a} uq \qquad G = \frac{\cos\phi}{a} vq$$

Assignment

- What fraction of pollution emitted in Colorado in January is in the Southern Hemisphere by the end of June?
- On a grid nlon=64, nlat = 32 construct a 2-d model in which wind is read in, and a constituent is advected on a sphere using a spectral method and spherical harmonics.
- Wind data is from NCEP reanalysis and available each 6 hours (I will provide this)
- Add a "source" as a known flux of tracer into the atmosphere from the grid point nearest Colorado.

The model

$$\frac{\partial q}{\partial t} = -\nabla \cdot \left(\vec{V} q\right) + S$$

S is a source flux.

Use a centered finite difference for time

Use spherical harmonics to represent the spatial structure.

As such, the spatial derivatives are known analytically

Implementing model code

- What is the state variable?
  (a series of complex coefficients)

- Read wind on a grid
- Obtain tracer field on a grid
- Calculate non-linear fluxes on a grid
  (*uq* and *vq*)
- Assign the surface fluxes (S)
- Convert fluxes to spectral form
- Compute flux divergence in spectral form
  (just the advection, as per last week)
- Time step the state variable (in spectral form)

Spherical harmonic synthesis

Spherical harmonic analysis

Remember to output state every so often (say, 6 hours?)

I will provide subroutines to work with spherical harmonics

Discussion questions

- How did you choose the diffusion coefficient?
- How does the diffusion change your answer?
- How does the frequency at which you update the wind change your answer?
- What do you expect happens to the tracer distribution after some long integration time?

Note, there will be no stub, but there will be examples of how to use the spherical harmonic transform subroutines