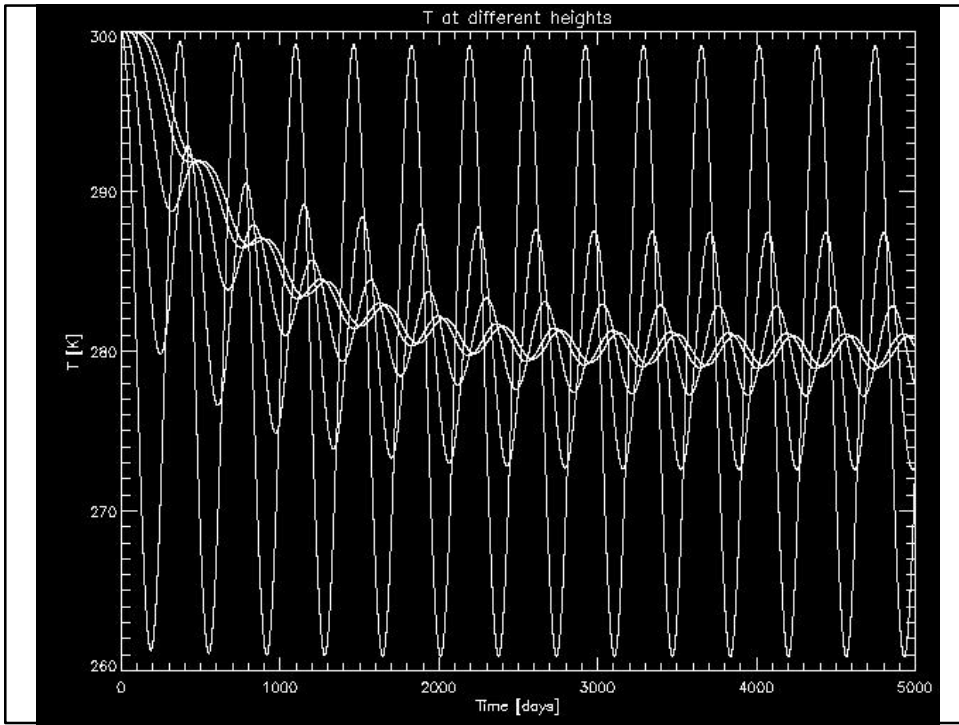
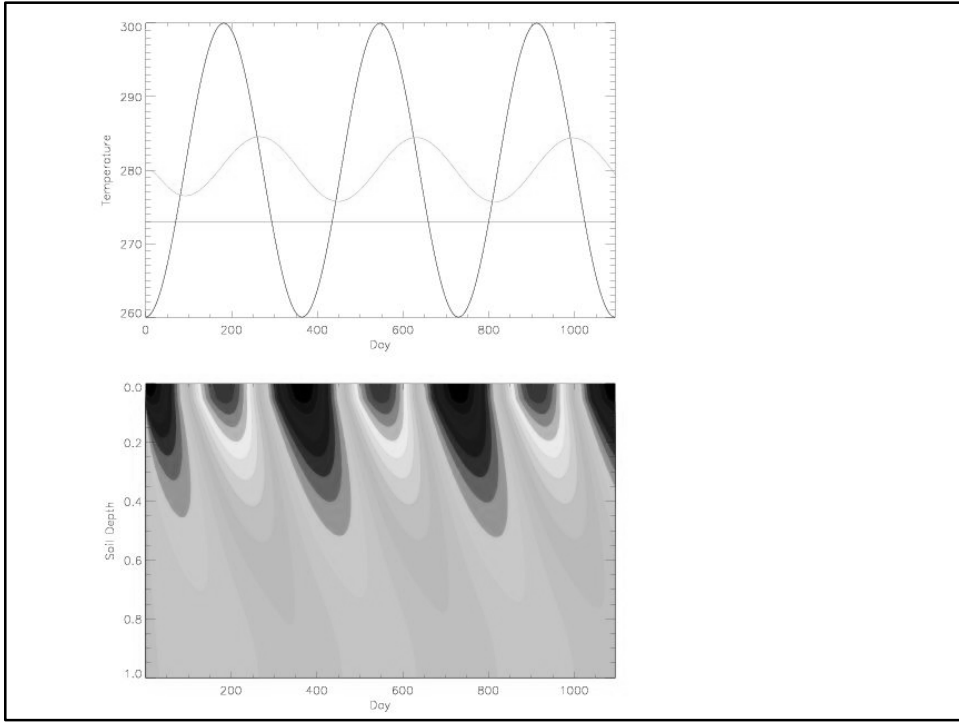
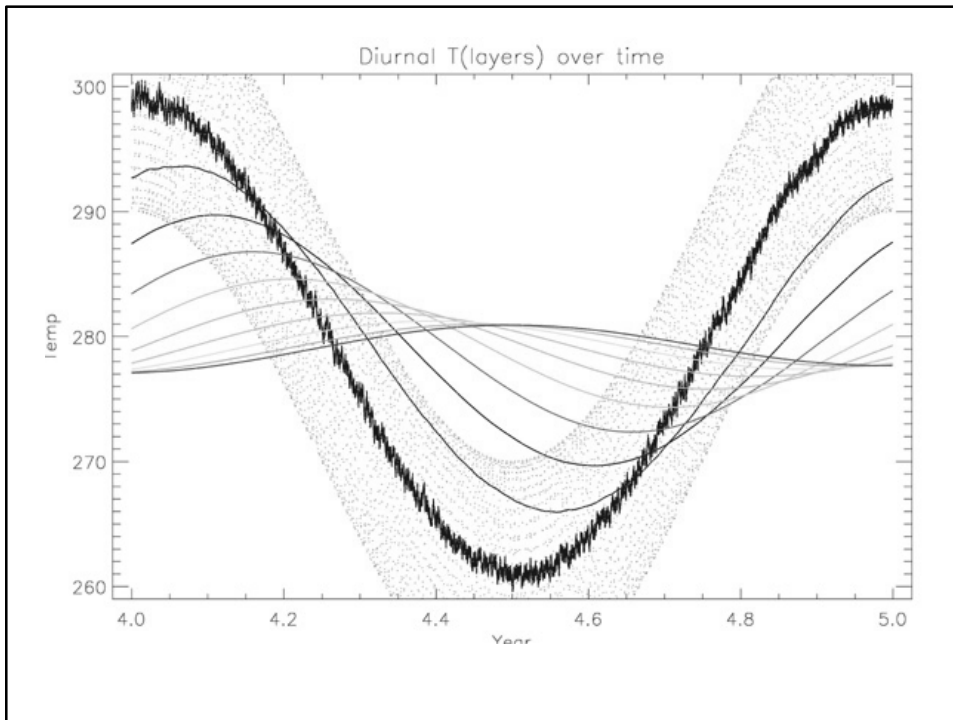


Vorticity and streamfunction

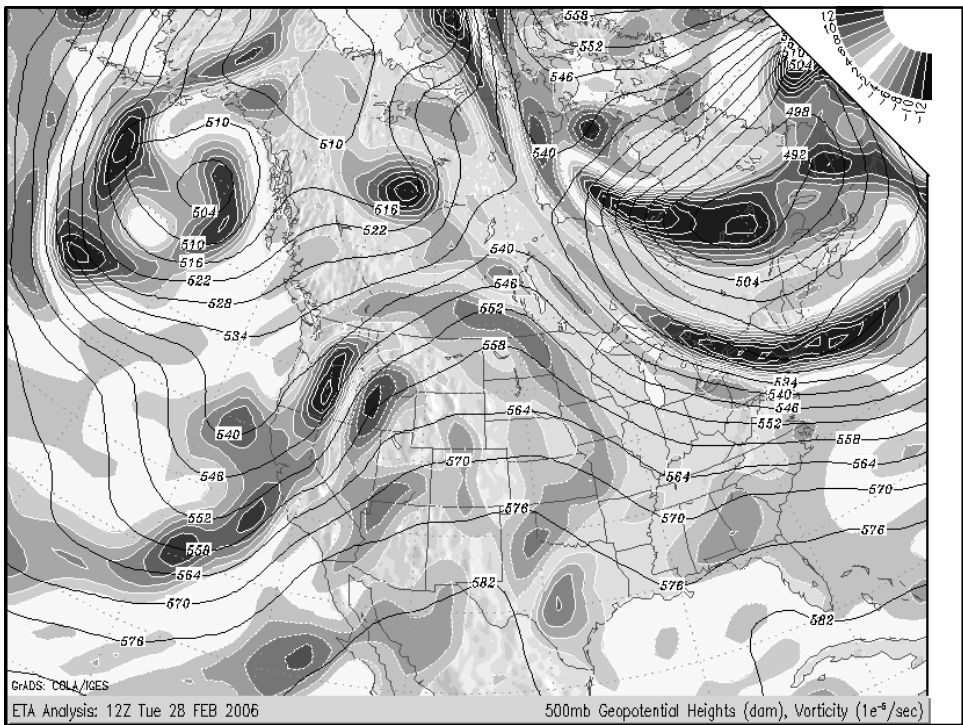
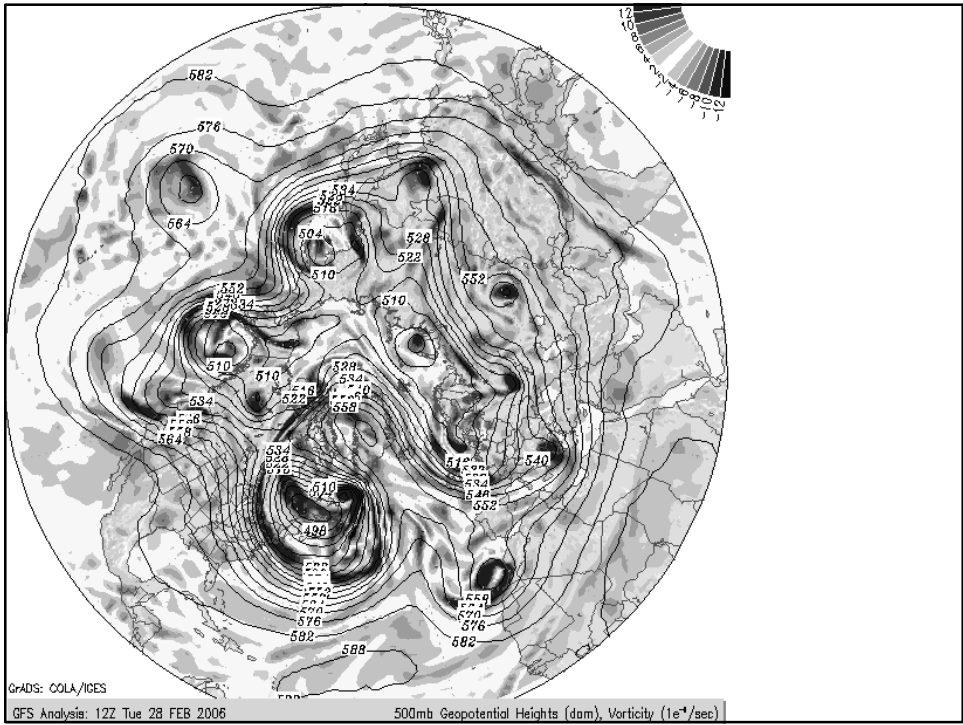
- How does the result change if I aim to plant on the other side of the yard where the soil is clay rich?
- How does the result change if we include a diurnal cycle ($Ad = 5K$)?
- What is the time mean soil temperature?
- Is the result the same for all time steps?
- Does the scheme conserve mass?
- Does the scheme conserve variance?
- Is the system positive definite?
- Is the phase and amplitude of the seasonal cycle the same at all depths?
- What is the role of the surface heat flux (and the magnitude of D) in the result?
- Do your predictions match the date given by the Farmers Almanac?





The 2d realm

- Advection in 2d?
- Diffusion in 2d?
- Can it be done with finite differences?



Vorticity advection

(absolute vorticity conserved)

$$\mathbf{h} = \mathbf{z} + f \quad \mathbf{z} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \quad f = 2\Omega \sin \mathbf{q} \approx f_0 + \mathbf{b} \cdot \mathbf{y}$$

$$\frac{d\mathbf{h}}{dt} = 0 \quad \frac{\partial \mathbf{z}}{\partial t} = -u \frac{\partial \mathbf{z}}{\partial x} - v \frac{\partial \mathbf{z}}{\partial y} - \mathbf{v} \cdot \nabla \mathbf{b}$$

$$\frac{\partial \mathbf{z}}{\partial t} = -\frac{\partial}{\partial x} [u(\mathbf{z} + f)] - \frac{\partial}{\partial y} [v(\mathbf{z} + f)]$$

Vorticity and stream function

- Stream function
(flow along contours)
- For geostrophic flow (and QG), we can thus deduce streamfunction from geopotential height

$$u = -\frac{\partial \mathbf{y}}{\partial y}$$

$$v = \frac{\partial \mathbf{y}}{\partial x}$$

$$\mathbf{y} = \frac{g}{f_0} \Phi$$

- Vorticity is the Laplacian of the streamfunction
(Thus streamfunction looks like a smooth or diffused version of vorticity)

$$\mathbf{z} = \nabla^2 \mathbf{y}$$

Assignment

- What will the weather be next Tuesday?
Make a 5-day weather forecast from Thursday using the non-divergent barotropic vorticity equation on a beta plane.
- Initialize with observed 500 hPa geopotential height
- Output predicted geopotential height, vorticity and u and v wind components.
- Choose a grid of $n_{lon}=64$ and $n_{lat} = 16$, with a 500 km spacing and a time step of 1 hour.
- Domain is cyclic in longitude, and bounded ($v = 0$) on the north and south boundaries.
- Choose boundary condition $\psi = \text{mean}$ at N and S edges.

Coding challenges

- Calculate streamfunction from geopotential
 - Calculate vorticity from for streamfunction
1. Given vorticity, obtain streamfunction
 2. Calculate u and v wind from streamfunction
 3. Predict evolution of vorticity (1 hour time step)
 4. Loop for 5 days

Code elements (suggested subroutines):

- Finite difference X and Y derivatives
- Finite difference Laplacian
- Finite difference Poisson solver (invert Laplacian)