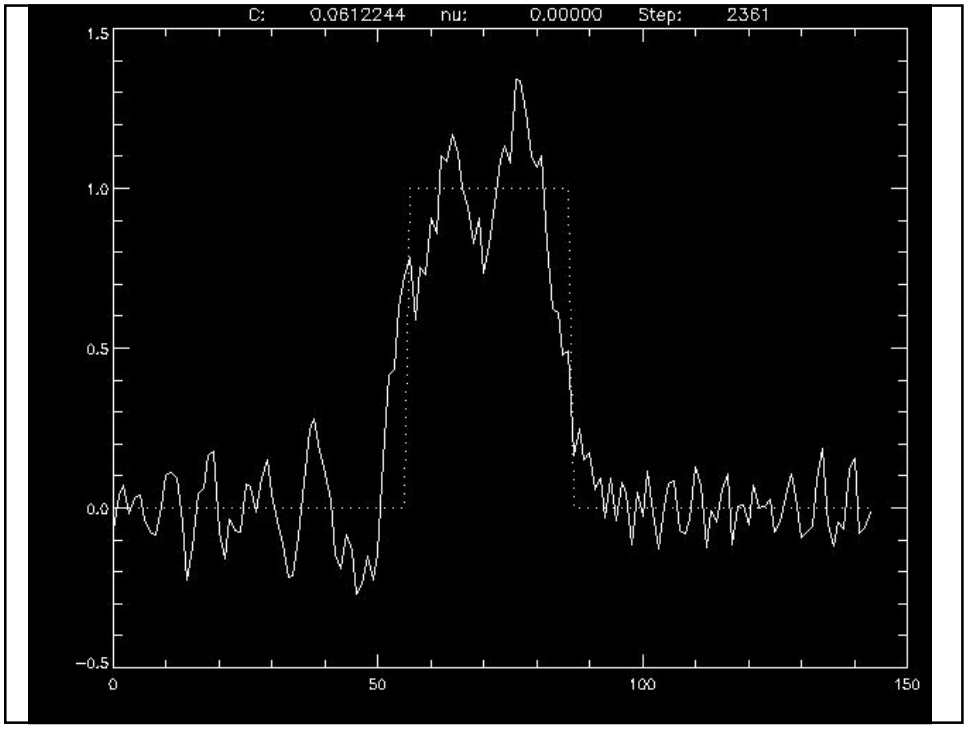
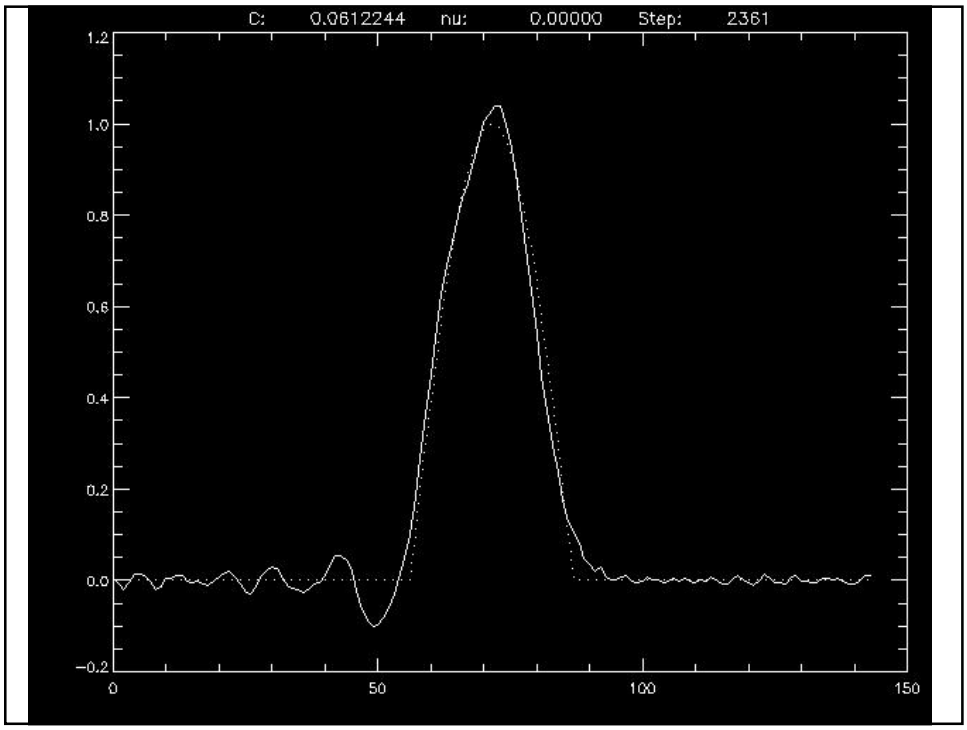
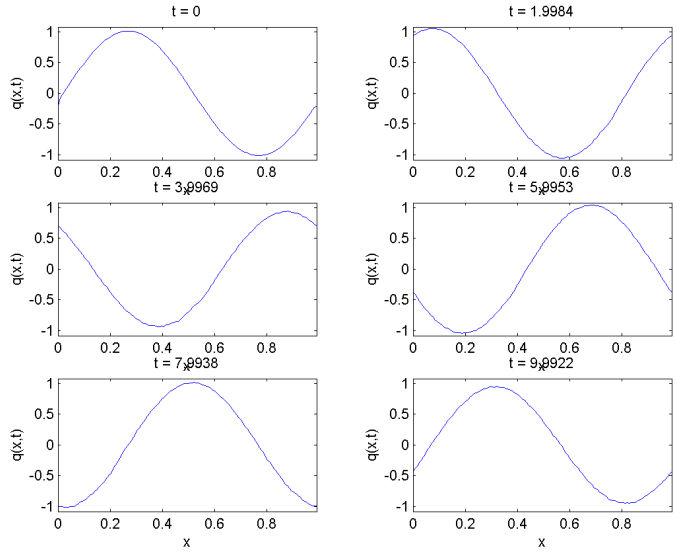
Advection (II)

Discussion

- Does the scheme conserve mass?
- Does the scheme conserve variance?
- Is the system positive definite?
- What is the critical Courant number stability?
- Is the forward time difference truly unstable?
- Do errors grow indefinitely for the stable system?
- Is an error in phase as much if a problem as an error in amplitude?
- Are errors dependent on the shape you are advecting?
- Can you separate the computation and physical modes?


C: 0.0612244 nu: 0.00000 Step: 2361
1.5
1.0
0.5
0.0
−0.5
0
50
100
150


C: 0.0612244 nu: 0.00000 Step: 2361
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
50
100
150

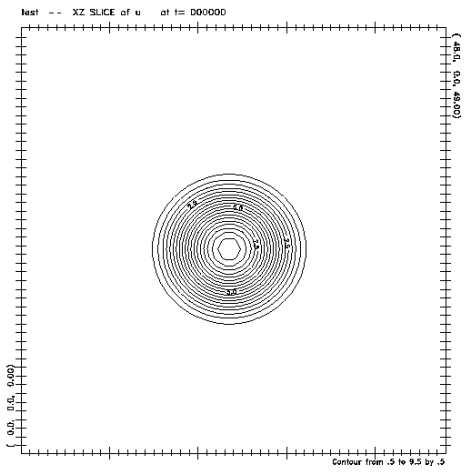


C = 0.9

Errors

- Amplitude errors
- Phase errors
- Dispersion error
  (needs more than one wave)

- Come about because superposition of individual modes behave difference
  (i.e., consider a Fourier decomposition)
- Also can appear as "numerical diffusion"
  (i.e., the scheme is dissipative)



http://www.atmos.ucla.edu/~fovell

Advection (second attempt)

- Even though centered in time centered in space differencing is stable, and converge as Δx and Δt become small there are still inaccuracies.

$$\frac{\partial q}{\partial t} \approx \frac{q_i^n - q_i^{n-1}}{\Delta t} = -u_i \frac{q_{i+1}^n - q_{i-1}^n}{2\Delta x}$$

•Problems come from either the time derivative, or the space derivatives because they are approximate.

•Let's live with the time problem, and work on the space.

•If we have a sine wave, we know the derivative exactly.

•If we have a sum of sine waves, we still know the derivative exactly!

Lab assignment

- **Part 1:** Modify your advection model from last week to use a Fourier decomposition to compute the space derivative..
- **Optional part 2**: Reformulate your advection model to use Fourier coefficients as the state variables.
- Contrast your results with those from the finite difference scheme. (i.e., contrast your error statistics). Try different time steps, shapes, number of Fourier coefficients used in the series.
- Comment on differences between amplitude, phase, and dispersion errors? Are there other errors?
- Set up a grid with 32 points from west to east such that one complete circle around that earth will occur every 20 days with u = 10 m/s. This should tell you the size of your Δx (and thus what latitude you are at!)