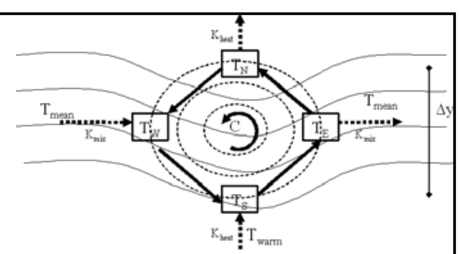
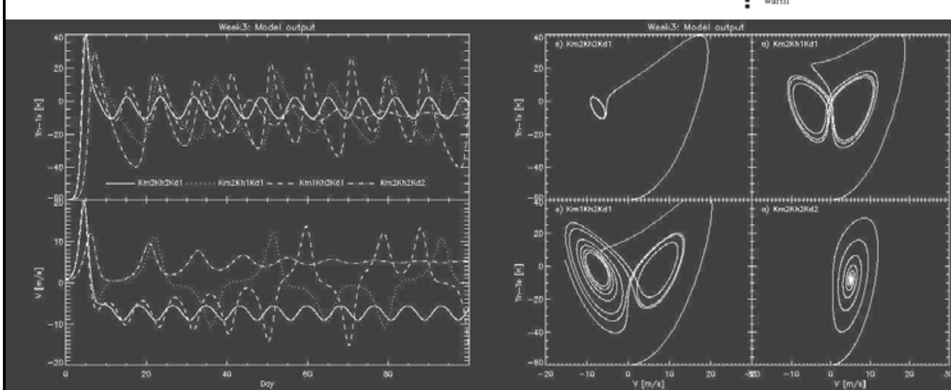
Advection


K_heat
T_N
T_mean
T_W
K_mix
C
T_E
T_mean
K_mix
Δy
T_S
K_heat
T_warm


Week3: Model output
Km2Kh2Kd1
Km2Kh1Kd1
Km1Kh2Kd1
Km2Kh2Kd2
Day
V [m/s]
Week3: Model output
V [m/s]
V [m/s]

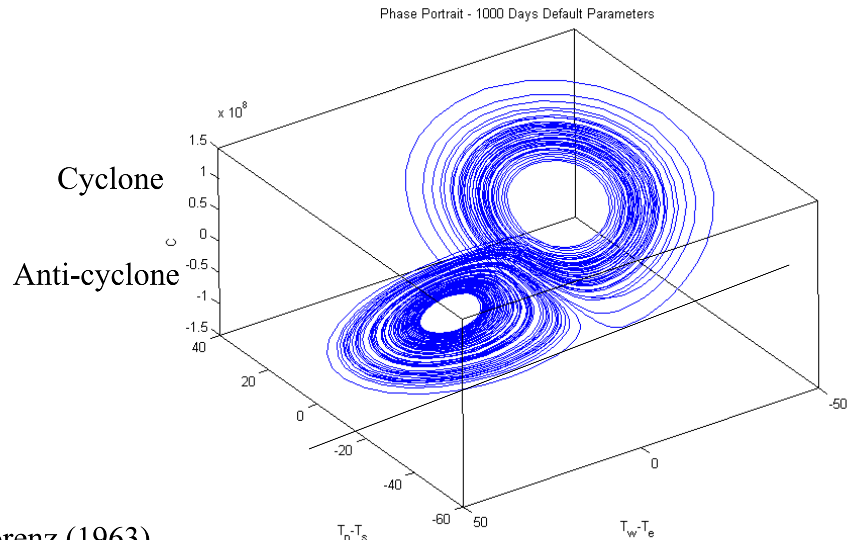
Discussion

- Is the initial condition balanced? Is there an initial "adjustment" period?
- What is a range of prediction (in days)? Does this depend on the initial conditions?
- What is the average (time mean) temperature difference? How does this compare to the radiative equilibrium temperature difference?
- Is the poleward heat transport always positive?
- What is the mean windspeed? *(Hint: consider the temporal variance in windspeed)*
- Are your results sensitive to selection of "free" parameters?

Weather and climate

In the model…

- how do you change the weather?
- how do you change the climate? (how do you DEFINE the climate?)



Lorenz (1963)

Low dimensional attractors. Unclear if they are a good analogy to the true climate system, but they have some appealing characteristics.

Non-linear advection

- Linear models have relatively simple behavior (consider an damped harmonic oscillator)
- We may infer that non-linearities make models interesting, and often require numerical methods
- The cyclone model has interesting behavior because of the non-linear advection. (product VT)
- Advection is one key aspect of hydrodynamic flows

Problems in time and space

(classes of 2nd order PDEs)

$$\frac{\partial^2 T}{\partial t^2} = c^2 \frac{\partial^2 T}{\partial x^2}$$ Wave equation (hyperbolic)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = f(x, y)$$ Poisson equation (elliptic)

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2}$$ Diffusion equation (parabolic)

$$\frac{\partial T}{\partial t} = -c \frac{\partial T}{\partial x}$$ Advection equation (first order)

Plus suitable boundary and initial conditions

These are the simplest canonical forms… they get nasty quickly

Advection

Consider the conservation of a quantity: $q = q(x,t)$

[could be CO_2, potential temperature, ozone, potential vorticity, water…]

$$\frac{dq}{dt} = S$$

S is some "source"

$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + \frac{dx}{dt}\frac{\partial q}{\partial x} = S$$

$$\boxed{\frac{\partial q}{\partial t} = -u \frac{\partial q}{\partial x} + S}$$

e.g., S = 0

1-dimensional advection equation ("one way wave equation")

More generally, $$\frac{\partial q}{\partial t} = -\vec{V} \cdot \nabla q + S$$

Finite difference schemes

Up-stream: Flux in – flux out

$$\frac{\partial q}{\partial t} \approx \begin{cases} -u(q_i - q_{i-1})/\Delta x & u > 0 \\ -u(q_{i+1} - q_i)/\Delta x & u < 0 \end{cases}$$

More formal finite differences

- Forward in time,
  centered in space

$$\frac{\partial q}{\partial t} \approx \frac{q_i^n - q_i^{n-1}}{\Delta t} = -u_i \frac{q_{i+1}^n - q_{i-1}^n}{2\Delta x}$$

- Centered in time,
  centered in space

$$\frac{\partial q}{\partial t} \approx \frac{q_i^{n+1} - q_i^{n-1}}{2\Delta t} = -u_i \frac{q_{i+1}^n - q_{i-1}^n}{2\Delta x}$$

One can of course construct many others….

Stability analysis

$$\frac{\partial q}{\partial t} = -u \frac{\partial q}{\partial x}$$

- Von Neumann method
- Consider known solution: q = A exp(i ϕ)
- Check how amplitude changes with time
  - If A continues to grow with time, then unstable
  - If bounded, then stable, but may not be accurate!
- Certainly errors in amplitude, but also consider phase errors.
- **Courant number C = |uΔt/Δx| < 1 for stability**
  (This is the CFL condition)

Courant-Fredrichs-Levy (CFL) condition

- Information can not move more than one grid box in one time step.

$$C = \frac{u\Delta t}{\Delta x} < 1$$

- Time it takes to move across one box is vΔx must be greater than the time step.
- Thus *maximum* speed of any stable motion is limited by the combination of Δx and Δt.

- Another interpretation: $q_i^{n+1} = (1-C)q_i^n + Cq_{i-1}^n$
- So advective translation is an *interpolation* in space. If C > 1, then it is an extrapolation, and susceptible to exponentially growing errors (amplitude grows each time step)

Lab exercise

- ***How accurate is the finite difference prediction?***

- Construct a 1-d model of linear advection (i.e., *u* = constant) to represent the zonal flow.
- Develop metrics of error
(consider phase and amplitude!)

- Do results change with wave number, time step, Courant number, shape of function, time filter?

Note this assignment will be useful for the mid-term assignment!