ATOC 7500: The Art of Climate and Environmental Modeling

Mid Term Assignment

Advection-diffusion model in one-dimension

Due: 4pm 10 March 2006

A general form for transport of some quantity, q, can be considered a combination of advection and diffusion:

$$\frac{\partial q}{\partial t} = -u\frac{\partial q}{\partial x} + K\frac{\partial^2 q}{\partial x^2}$$

Such an equation occurs naturally in many environmental applications from large scale motion of the atmosphere, growth of ice crystals in clouds and percolation of water through soil to name a few. Build two models for solving this equation: 1) a finite difference grid point model, and 2) a spectral model with Fourier coefficients. *Quantitatively* compare the behavior of your two models.

This is an open ended project and can be as detailed or as concise as you see fit.
(It is permissible, for instance, to consider the simpler case where $K = 0$.)

Your report must address by way of comparison between the two models:

- Explanation of the physical system to which this equation is being applied
- Design and description (with a justification) of the numerical formulations
- Definition and demonstration of the numerical stability of the system
- Discussion of amplitude errors, phase errors, and dispersion errors
- Definition and discussion of metrics used for quantitative comparisons

Make sure you fully describe your model experiments in the context of, first, a hypothesis to test, and second, the details of the model configuration. In particular, you should describe what values are chosen for which parameters, and why this selection is applicable to the hypotheses you are testing. It is likely that you will produce many, many graphs of output. You only need to include those which are most appropriate. All figures should be accompanied with captions of sufficient detail and referenced in the text of your report. Some very detailed aspects of your report might appear as appendices.

Your report might also include:

- Discussion of positive definiteness, conservation of mass and energy
- Importance of boundary conditions (cyclic, or otherwise)
- Definition and investigation of the computational and physical modes
- Inclusion of non-linear effects (i.e., q = u)
- Other numerical methods (finite volume or semi-Lagrangian schemes, etc.)
- Physics (or chemistry!) missing in your model

The specific goals addressed in your report should be stated in an introduction, a methods section should outline the experiments, and conclusions should be given to summarize results in a conclusions section.

The modeling tasks can be completed individually or in small groups, but your report and the focus of your experiments will be your own.