ATOC 7500: The Art of Climate and Environmental Modeling

Week 10 Lab Assignment

General circulation model of the atmosphere

The general circulation of the atmosphere is responsible for balancing the excess heat in the tropics with the heat deficit in the polar regions via atmospheric motion. This motion is characterizsed in the mid-latitudes by almost geostrophic flow. As such, we can use quasi-geoistrophic (QG) theory as a basis for a general circulation model. The model must include an energy source (from the sun) and energy sinks (thermal radiation and dissipation) to sustain a circulating climate. We also require a baroclinic atmosphere to allow generation of synoptic scale eddies. As such, we need vertical structure, and specifically, the ability to resolve the stretching term in the vorticity equation.

The QG potential voticity (PV) is written (assuming a constant thermal stratification) as $Q = q + f$ where q is the relative PV:

$$q = \zeta + \frac{f_0^2}{\sigma}\frac{\partial^2 \psi}{\partial p^2}$$

and we recall for QG flow (non-divergent) $\zeta = \nabla^2 \psi$. Thus prediction can be made as given conservation of QG PV as:

$$\frac{\partial q}{\partial t} = -V \cdot \nabla(q + f) + K\nabla^2 q - D\zeta + H$$

Here we have added drag, and forcing by some (known) diabatic heating H. Notice the drag is due only to the vorticity (not the PV). Again, if the wind images U and V are known, we can evaluate the advection as $-\alpha(F, G)$ with *F=U(q+f)* and *G = V(q+f)*. Similarly we can write the drag as $D\zeta = \alpha(DV, -DU)$. Thus the challenge is to obtain U and V from *q*.

We apply a finite difference discretization in the vertical, such that for level "k" we can write for each spherical harmonic coefficient:

$$q_n^m(k) = \nabla^2 \psi_n^m(k) + \frac{1}{L^2}\left(\psi_n^m(k-1) - 2\psi_n^m(k) + \psi_n^m(k+1)\right)$$

L is the Rossby radius (taken to be around 600 km). Thus knowing the relative PV (q), we can invert this elliptic function to obtain the streamfunction. Knowing the streamfunction, we can thus trivially compute the vorticity, and ultimately the wind speeds U and V. Rewriting, and making use of the spherical harmonic Laplacian operator, we find this desctribes a tridiagonal matrix of the form:

$$\left[\frac{1}{L^2}\right]\psi_n^m(k-1) + \left[\frac{-n(n+1)}{a^2} - \frac{2}{L^2}\right]\psi_n^m(k) + \left[\frac{1}{L^2}\right]\psi_n^m(k+1) = q_n^m(k)$$

which is easily solved with a tridiagonal matrix solver.

Thus all parts of the time derivative are known, can we can perform the time marching simulation using the normal finite difference approximation in time for spectral coefficients (indicies m and n dropped for simplicty):

$$q^{next} = q^{last} + 2\Delta t\left[-\alpha(F,G)^{now} - \alpha(DV,-DU)^{last} + K\left(\nabla^2 q\right)^{last} + H\right]$$

A prediction can be made:

1) Start with the spectral field (q)
2) Invert the PV field to obtain streamfunction (ψ), and this vorticity (ζ)
3) Evaluate the wind images U and V from the vorticity (ζ) in spectral space
4) Transform q, U and V to a grid (spherical harmonic synthesis)
5) Compute fluxes F and G on a grid
6) Compute drag terms on a grid (only for lowest level!)
7) Evaluate solve "alpha" to get spectral flux divergence
8) Compute the Laplacian for diffusion
9) Construct time derivative as the sum of parts
10) Step the spectral coefficients of vorticity, and update
11) Loop to 2

Construct a model with two layers (nlev=2). Set up a computational grid nlon=64, nlat = 32 and (ntrn=21, ltriang=.true.). Use as 0.5 hour time step (48 steps per day) to make a 100 day climate simulation prediction (start with 1 day!). Output results only once per day.

You might start with the NDBVM from last week, and make it "n" un-coupled layers. The addition step needed here is to provide the vertical coupling via solution of the elliptic equation for relative PV. (Hint, you can solve the tridiagonal system for real and complex parts separately!)

Discussion question

- How do you define the climatology?
- In what way does the simulation look like the real general circulation? In what ways does it not?
  (westerly jets? Hadley cells? Ferrell cells? tropical easterlies?)
- How do you determine if the model is "spun up"?
- How does the behavior change with "tunable" parameters:
  - Rossby radius
  - Diabatic heating rate
  - Drag coefficient

  (comment on the physical interpretation of these changes)
- How can you include topography (hint, see last weeks model)? How does this change the simulation?
- What are some ways to better choose the forcing?
- How would you expand this model to be a fully fledged "climate" model?