ATOC 7500: The Art of Climate and Environmental Modeling

Week 9 Lab Assignment

Spectral vorticity weather prediction model

What will the weather be next Tuesday? Although we have a global model, let's focus on continental US.

Recall (from Week 7 Lab), that we can write the the evolution of the atmosphere via a conservation of potential vorticity on a sphere (with radius a, longitudes λ and latitudes ϕ in radians):

$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + \cos\phi \frac{u}{a}\frac{\partial q}{\partial \lambda} + \frac{v}{a}\frac{\partial q}{\partial \phi} = 0$$

For a non-divergent atmosphere, this is just conservation of angular momentum ($q = \zeta + f$) , which can be written in terms of vorticity alone:

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial}{\partial \lambda}\left[\cos\phi \frac{u}{a}(\zeta + f)\right] - \frac{\partial}{\partial \phi}\left[\frac{v}{a}(\zeta + f)\right]$$

After discretization, we obtain our prediction equation:

$$\frac{\partial \zeta}{\partial t} = -\alpha(F,G) + k\nabla^2\zeta - D\zeta$$

This model includes transport by the resolved motion (via the α divergence operator), transport by unresolved motions (via the diffusion). We also include a linear drag term to represent the exchange of momentum between the atmosphere and the ground (See Holton 5.4).

Recall the advection can be resolved exactly using spherical harmonics, where,

$$\alpha(F,G) = \frac{1}{\cos^2\phi}\left[\frac{\partial F}{\partial \lambda} + \cos\phi \frac{\partial G}{\partial \phi}\right]$$

and F = Uq, and G = Vq, where $U = \frac{\cos\phi}{a}u$ and $V = \frac{\cos\phi}{a}v$ are the so-called wind images. Notice by dividing by *a*, we no longer need to know the radius!

We know vorticity is related to streamfunction, and the for a non-divergent flow, the velocities, u and v are just the gradients of the streamfunction. Using spherical harmonics, we can invert the Laplacian of streamfunction to obtain voticity by a simple division by $-n(n+1)$:

$$\zeta = \nabla^2\psi \text{ so } \zeta_n^m = \nabla^2\psi_n^m = -n(n+1)\psi_n^m$$

Similarly we can evaluate the gradieients using the well known recurrence rtelationships (see lecture notes), and as such evaluate wind fields as:

$$U_n^m = \frac{1}{n(n+1)}\left[-im\delta_n^m - (n+1)\varepsilon_n^m\delta_{n-1}^m + n\varepsilon_{n+1}^m\delta_{n+1}^m\right]$$
$$V_n^m = \frac{1}{n(n+1)}\left[-im\delta_n^m + (n+1)\varepsilon_n^m\zeta_{n-1}^m - n\varepsilon_{n+1}^m\zeta_{n+1}^m\right]$$

where δ is the divergence, and zero in our case (i.e., non-divergent). Thus knowing the vorticity, we can directly obtain U and V from the appropriate wave number manipulations. Notice also that for V at wave number n, we require vorticity at wave number n-1, and n+1. As such, we must have one extra wave number (n+1) for the calulation of wind if we are to be able to recover the vorticity from the wind field exactly.

Thus a prediction can be made:

1) Start with the spectral ζ field
2) Evaluate the wind images U and V from the vorticity in spectral space
3) Transform ζ, U and V to a grid (spherical harmonic synthesis)
4) Compute fluxes F and G on a grid
5) Evaluate solve alpha to get spectral divergence (spherical harmonic synthesis)
6) Compute the Laplacian, drag to form the spectral time derivative
7) Step the spectral coefficients of vorticity, and update
8) Loop to 2

Set up a computational grid nlon=64, nlat = 32 and (ntrn=21, ltriang=.true.). Use as 0.5 hour time step (48 steps per day) to make a 5 day prediction. Output results only every 6 hours.

- It may be helpful to start with an advection model with fixed winds (e.g., last week's assignment), and convert this to a vorticity model, by adding a step to compute U and V.
- Also, we know that a single spherical harmonic is an exact solution (the so called Rossby-Haurwitz wave (see Holton 13.5.2). If we can correctly model this, we can have confidence in the numerical method.

Discussion question

- What range of values for *K* seem reasonable?
- Does drag help? What values of *D* seem reasonable?
- How different is your prediction if the initial condition are slightly different? (Try adjusting the initial spherical harmonic coefficients by some small fraction)
- Can this initial condition modification help **quantify** the possible forecast error? (Can you come up with a number?)
- Does adding mountain topography help?
  Mountains can be added by adjusting our definition of potential vorticity such that
  $$q = \zeta + f\left(1 + \frac{h}{H}\right)$$
  where h is the height of earth's topography, and H is the scale height (~7600m)
- In what ways is this model better than the grid point model we built?