ATOC 7500: The Art of Climate and Environmental Modeling

Week 8 Lab Assignment

Spectral tracer model

What fraction of pollution emitted in Colorado in January is in the Southern Hemisphere by the end of June?

Motivated by the need to improvde the representation of advection, and the need to remove boundary conditions, we want to build a model for advection of some passive tracer on a sphere. The 2d model is

$$\frac{\partial q}{\partial t} = -\alpha(uq, vq) + k\nabla^2 q + S$$

S is the net sources and sinks and we have a divergence operator on a sphere:

$$\alpha(A, B) = \frac{1}{\cos^2\phi}\left[\frac{\partial A}{\partial \lambda} + \cos\phi\frac{\partial B}{\partial \phi}\right]$$

In the spectral method we replace all variables with a harmonic series of the form:

$$q(\lambda, \phi) = \sum_m \sum_n q_n^m P_n(\mu) e^{im\lambda}$$

where the basis functions are spherical harmonics $Y_n^m = P_n(\mu)e^{im\lambda}$. Spherical harmonics and the associated Ledendre polynomials have a varietry of unsfile recurrance relations that enable us to deduce the spatialderivatied of each harmonic perfectally, and thus reolve the advection perfectly. Of particular note is

$$\nabla^2 q_n^m = -\frac{n(n+1)}{a^2} q_n^m$$

which is reminicent of the fact we required our basis functions to be eigenfunctions that are solutions of Laplace's equation on a sphere.
The state variable in this model is thus the series of (complex) spherical harmonic coefficients q_n^m . Given a centered finite difference approximation we can march forward in time, in the usual manner.

Set up a computational grid nlon=64, nlat = 32 and (ntrn=21, ltriang=.true.). Use as 0.5 hour time step (48 steps per day), and output results only every so often. Start with simulations of only a few days in length.

Tips for implementation

1. Set up time stepping framework, and output file (perhaps copy this from last weeks code)
2. Set number of total steps to 1 for testing
3. Set some initial qg. convince yourself you can to forward and backward transforms – plot them.
4. Set some simple wind field (say solid body rotation)
5. Compute and transform “F” and “G” and try a few steps
6. Add diffusion
7. Add source term (on grid, convert to spectral)

- Try reading observed wind field
  (you will need to use horizontal diffusion to he useful results)
- Keep the model as simple as possible until you are convinced it works.

Discussion questions

- How did you choose the diffusion coefficient?
- How does the diffusion change your answer?
- How does the frequency at which you update the wind change your answer?
- What do you expect happens to the tracer distribution after some long integration time?