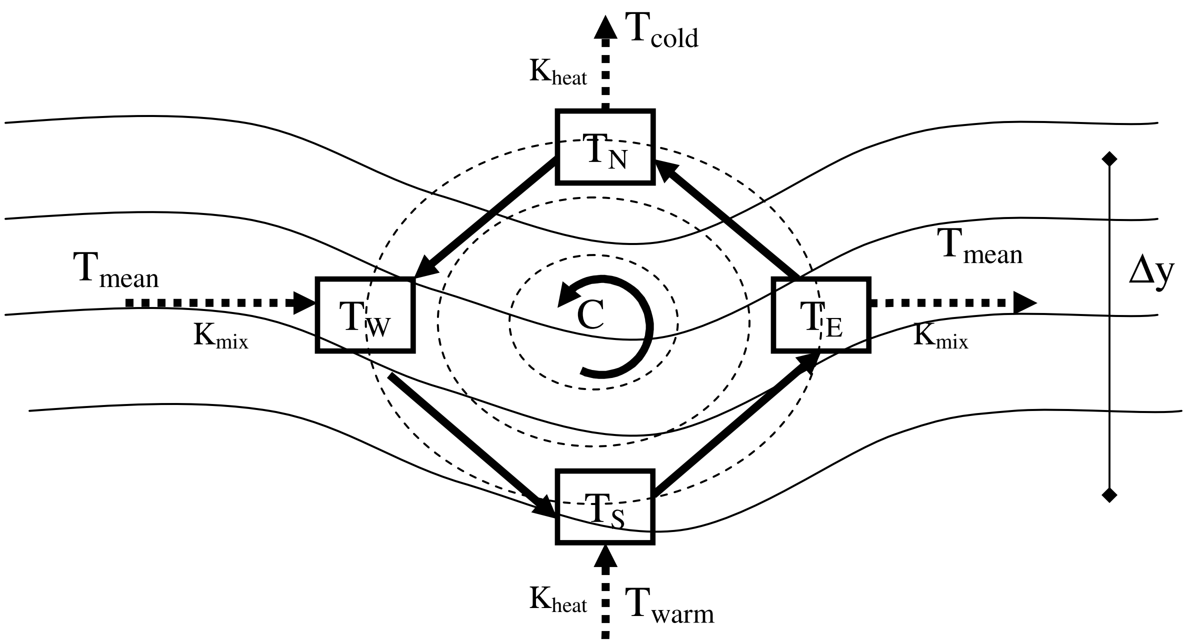
ATOC 7500: The Art of Climate and Environmental Modeling
Week 3 Lab Assignment
Mid-latitude cyclones and heat flux

It is typical in energy balance models to include meridional structure via multiple boxes between which heat is exchanged via a heat flux parameterized as a diffusive type process. Such a scheme, while providing satisfactory results (e.g., McKuffie and Henderson-Sellers, chapter 3), is inadequate in capturing the essential physical mechanism. An alternate approach can be constructed using well known atmospheric dynamics. Specifically, we can model the heat transport as the sum of northward warm advection and southward cold advection along the sides of a rotating asymmetric cyclone. For a 1-layer atmosphere (by which the zonal mean meridional velocity vanishes), the heat flux may be written as F = -d(v'T')/dy or simply sum(vT)/dy. The change spin rate of the cyclone, from which the wind speed can be deduced, is due to the imbalance between acceleration associated with baroclinic instability frictional dissipation.

Consider a cyclone embedded in a background zonal flow (thus producing a "wave" type pattern), with temperatures on the north, south, east, and west edges (T_N, T_S, T_E and T_W). The circulation is an integral measure over the region of the cyclone with length scales Δy.



This configurtation allows recharge of the air within the cyclone by mixing to the background zonal mean temperature. The system is driven by energy input on the warm southern side and loss of energy on the cold northern side.

We can evaluate the advection term knowing the windspeed (assumed constant around the cyclone) and the temperature at the midpoints between boxes. That is, the temperature change is simply the difference between heat flux in minus heat flux out, per unit distance. The flux between to adjacent boxes, say N and W, may be written as:

$$F_{N-W} = \frac{2V}{\Delta y}\frac{(T_N + T_W)}{2} \qquad (1)$$

Where *V* is the tangential velocity. The heating and cooling to the north and south can be represented as a simple relaxation to a radiative equilibrium profile (i.e., T_{cold} and T_{warm}) with a rate given by K_{heat}. Similarly the mixing between air inside and outside the cyclone can be assumed to occur with a rate constant K_{mix} and relax the temperature to the zonal mean: $T_{mean} = (T_{warm} + T_{cold})/2$. As such, we can finally write the temperature budgets as:

$$\begin{aligned}
\frac{dT_N}{dt} &= F_{E-N} - F_{N-W} + K_{heat}\left(T_{cold} - T_N\right) \\
\frac{dT_W}{dt} &= F_{N-W} - F_{W-S} + K_{mix}\left(T_{mean} - T_W\right) \\
\frac{dT_S}{dt} &= F_{W-S} - F_{S-E} + K_{heat}\left(T_{warm} - T_S\right) \\
\frac{dT_E}{dt} &= F_{S-E} - F_{E-N} + K_{mix}\left(T_{mean} - T_E\right)
\end{aligned} \tag{2}$$

Notice that the advection only moves heat around, while the heating and mixing terms provide zonal and meridional variations that give rise to baroclinicity.

To close this system, we need also to compute the tangential velocity *V*. This is conveniently found via the circulation:

$$C = \oint V \cdot dl = 4V\Delta y \tag{3}$$

Circulation increases due to a gravitational torque associated with variations in density along a pressure surface. That is, if there is a lateral displacement of air along some slopped trajectory that places low density air beneath high density air, there will be convective-like overturning. As such, from the circulation theorem we may write the effect of this buoyant torque as a temperature difference (since temperature is proportional to density for an ideal gas) between the background thermal structure, then combined for both the northerly and southerly air streams:

$$\frac{dC}{dt} = g\left(\frac{T_E - T_W}{T_{mean}}\right)S\Delta y - K_{drag}C \tag{4}$$

S is the slope of the displacement, which must be assumed, and g is the acceleration due to gravity. Notice the addition of a linear damping term on the right that captures the effects of surface friction, and spin-down of the cyclone. In this formulation we model a meridional overturning (i.e., a residual circulation) that is in thermal wind balance, but realize most of the motion is in fact in the horizontal plane due to stable thermal stratification and low Rossby number.

By choosing K_{heat}, K_{damp} and K_{drag}, and assigning a basic state thermal structure on which our cyclone will exist and draw energy, we have a closed set of equations which can be solved numerically using a filtered centered difference scheme.

For background reading revise Holton Chapter 4 (circulation), 6 (baroclinic instability), and 10 (role of heat flux, and asymmetry of waves/cyclones).

Parameter suggestions

g	$= 9.8$ m s^{-2}	
Δy	$= 2x10^6$ m	
Δt	$= 1800$ s	(0.5 hour time steps)
K_{drag}	$= 1x10^{-5}$ s^{-1}	(~ 1 day)
K_{heat}	$= 2x10^{-6}$ s^{-1}	(~ 6 days)
K_{drag}	$= 2x10^{-6}$ s^{-1}	(~ 6 days)
T_{warm}	$= 280$ K	
T_{cold}	$= 220$ K	
S	$= 0.0005$	(dz/dy for displacements)

Tasks

- Build the model to output the wind speed, T_N, T_S and the net heat flux for 100 day simulations
- Plot temperature difference (T_N-T_S) as a function of time
- Plot wind speed as a function of time
- For the 2 plots above, overlay additional curves from different runs in which the initial conditions vary.
- From one run plot temperature difference as a function of windspeed.

Tips/suggestions for implementation

- Start by setting up time stepping code with the 5 variables
- Evaluate the windspeed from the circulation
- Evaluate four advective fluxes, and heating and mixing terms so as to be able to perform one time step.
- Check model works for the first few steps
- Check model works for longer simulations

Question for discussion and experiments:

1) Is the initial condition balanced? Is there an initial "adjustment" period?
2) What is the average (time mean) temperature difference? How does this compare to the radiative equilibrium temperature difference?
3) What is the mean windspeed? (Hint: consider the temporal variance in windspeed)
4) What is a range of prediction (in days)? Does this depend on the initial conditions?
5) Are your results sensitive to selection of "free" parameters?
6) Is the poleward heat transport always positive?

Note, those feeling daring, this model can actually be written in three parameters, C, T_N-T_S and T_E-T_W!