ATOC 5060: Atmospheric dynamics

Homework assignment 3
Due 4pm Friday 21 March

Where moving fluid contacts stationary surfaces there is a frictional, or drag, force. In the atmosphere we think of the planetary boundary layer, while in the ocean we think of the mixed layer. In the rotating tank, friction at the inner and outer walls and at the lower boundary all contribute the overall frictional forces acting, and it is the friction forces that ultimately add or remove momentum from the fluid allowing it to spin up and spin down.

We use heated thermistors to measure the velocity of the fluid (which itself is a measurement that is based on the drag in the region of the probe!) and we wish to confirm two properties that emerge in cases with drag. They are placed at radial position r_1, closer to the outside, and r_2 closer to the rotation axis.

Experiment 1: Determine the equilibration time scale of the tank

Holton shows the evolution of relative vorticity with a barotropic fluid can be expresses as

$$\zeta(t) = \zeta(0)\exp\left(-\frac{t}{\tau}\right) \tag{1}$$

where t is the time, and τ is a time scale (which we'll called the equilibrium time scale) for the adjustment, and has units of seconds. To evaluate the actual value of τ, we need to know the true evolution of ζ.

a) The heated thermistors measure a voltage V that is proportional to the velocity u. A first step is to calibrate the measurements such that we obtain velocity values. At the instant of spin up, you can assume that the fluid is at rest, while the tank (and probes) are moving through it. For this instant use the measured rotation rate and calculate the tangential velocity at the radius of the two probes, and thus determine the linear calibration coefficients a and b for the two probes.

$u_1 = a_1 V_1 + b_1$ and $u_2 = a_2 V_2 + b_2$

b) From the spin down experiment, make a graphs of the evolution of the velocities u_1 and u_2. Make a graph of u_1 and u_2 over the duration of the experiment.

c) Using your data, calculate the vorticity ζ, and fit a curve of the form given by (1) to your experimental data and thus determine τ as that of the best fit. Make a graph showing the experimental ζ and your best fit curve. (Hint: if the take the log of (1), you find a linear relationship in time, where the "slope" of the line is proportional to τ. Also, be somewhat selective with the experimental data to find the curve which is the best fit!).

d) Repeat the calculation to determine τ for the case of spin down to from high to low rotation rates. Are they the same? Give a hypothesis as to why this might be the case.

e) Find an alternate definition for τ in terms of a viscosity. Evaluate τ for the case where only (molecular) viscosity is acting, and compare this you're your experimentally determined value. By computing the effective "eddy viscosity", explain why these differ, and, specifically, describe how many times more or less efficient turbulent eddies are at transporting momentum.

Experiment 2: Velocity profiles in the present of drag

One can write the drag force (or, momentum flux) as F = [u'v'] = $C_d u^2$, where C_d is the drag coefficient. We wish to compare the size of this force to that of the centrifugal force. To do so we must determine C_d, and we can make use of the velocity profile that emerges in the radial direction. i.e., u = u(r), where r is the distance inward from the outer wall.

We can also define the flux in terms of the friction velocity u*, and thus also define F = = $(u^*)^2$. If the momentum flux is approximately constant over some fraction of the tank, then we can integrate over the radial direction, to obtain a velocity profile:

$$u(r) = \frac{u^*}{k}\ln\left(\frac{r}{r_0}\right) \qquad (2)$$

where k is the von Karman constant (k=0.4) and r_0 is a dimensional measure of the "roughness" of the side wall surfaces (like z_0 found in text books). With the probes closer to the outer wall, we are able to deduce the radial velocity profile experimentally to determine u*, and thus F. We place the probes nearer to the outer wall to capture this profile.

a) Rewrite (2) for the difference $\Delta u = u_1 - u_2$ and thus determine an average u* for the duration of the experiment (say, 3 e-folding times).

b) Compute the frictional force at the two probe positions (r_1 and r_2), and compare them to the size of the centrifugal force at each position (express answer as a ratio). Can you ignore friction effects at r_1? What about r_2?

Tank data

Mean fluid depth:	H= 5 cm	
Inner radius:	R_a= 5.1 cm	
Outer radius:	R_b= 10.6 cm	
Radius of probes at setting 1:	r_1= 2 cm	r_2= 3.5 cm
Radius of probes at setting 2:	r_1= 0.2 cm	r_2= 1.7 cm
Kinematic viscosity of silicon oil:	K = 8×10^{-6} m^2/s	

Note: You will need to record the changing rotation rate, Ω, as the experiments progress.