ATOC 5060: Atmospheric dynamics

Homework assignment 2

Due 4pm Thursday 6 March

1. The time and space varying amplitude of velocity perturbations associated with a two-dimensional inertial gravity wave have the form $w^* = \mathrm{Re}\{W \exp(i\phi)\}$ and $u^* = \mathrm{Re}\{U \exp(i\phi)\}$, where $\phi = kx + mz - \nu t$, and W and U are complex amplitudes.

a. Write down a (linearized) continuity equation for the perturbation velocity field.
b. Using your continuity equation, express the U as a function of W.
c. Derive an expression of the mean vertical flux of westerly momentum ($\rho[u^* w^*]$), in terms of the mean amplitude of the vertical velocity perturbation. Assume density is constant, and [] denotes the spatial mean.
d. Under what conditions is this flux negative? (Hint: physical explanation of signs of *m* and *k*).
e. Is the flux larger or smaller for waves differing in each of only *m* or *k*?
f. If the wave propagates upwards and eastward, explain what effect the wave would have on a westerly jet should it break in the region of the jet and deposit the momentum it carried.

2. For a large scale atmospheric wave (single zonal wave number) which has correlated perturbations in both northward velocity and temperature (v^* and T^*), prove that the zonal mean (integral over x) northward flux can be written as

$$[v^* T^*] = \frac{1}{2}\mathrm{Re}\{V\hat{T}\}$$

V and T are complex amplitudes (i.e., $T = T_r + iT_i$) and "hat" denotes the complex conjugate. (Hint: careful with algebra on the l.h.s., the integral of $\cos^2 x$ = ½, and the integral of sin 2x=0)

3. One can define any quantity such that it has a mean component and a deviation from this mean. The mean can be defined in both space and time. As such, some quantity A is written

$$A(x,t) = \overline{[A]} + \overline{A(x)^*} + A'(x,t)$$

Where square brackets denote the zonal mean, the asterisk denotes the deviation from this, the overbar denotes a time mean, and the prime the deviation from this. Given this definition, show that one can write the zonal mean of the total heat transport as the sum of three terms: transport by the time mean and zonal mean flow, transport by stationary eddies, and transport by the transient flow (which one can assume is dominated by transient eddies). That is,

$$[\overline{vT}] = [\bar{v}][\bar{T}] + [\bar{v}^* \bar{T}^*] + [\overline{v'T'}]$$

4. Using the results from 2 and 3 above, construct a graph of zonal mean eddy heat transport at 45°N as a function of wave number at 500 hPa, 100 hPa and 10 hPa based on the NCEP reanalysis data for January 2007. Your 3 graphs should have lines for both stationary and transient eddies, and be aesthetically pleasing (meaning, labeled!). Summarize you findings: which planetary wave numbers are most influential in the heat transport at each pressure level?

Bonus question: Compare with July and the southern hemisphere cases. (Not graded)

You will need you to figure out Fourier transforms, like the IDL function FFT. See example on web site, and documentation at http://idlastro.gsfc.nasa.gov/idl_html_help/FFT.html