

## ATOC 5060: Atmospheric dynamics

Homework assignment 1

*Due 4 pm Tuesday 12 February,*

Problems can be worked on in groups, however, what you hand in should be your own work.

**1)** The flow in the midlatitudes is almost geostrophic, however to retain the time rate of change, we can not assume that it is totally in geostrophic balance. Starting with the  $u$  and  $v$  momentum equations in rectangular and isobaric coordinates (1), use only the first order approximation for the Coriolis parameter, and apply scaling arguments to derive an approximate  $u$  and  $v$  momentum equation:

Isobaric Cartesian:

$$\begin{aligned} \frac{du}{dt} &= fv - \frac{\partial \Phi}{\partial x} \\ \frac{dv}{dt} &= -fu - \frac{\partial \Phi}{\partial y} \end{aligned} \quad (1)$$

Approximate:

$$\begin{aligned} \frac{du_g}{dt} &= fv_a + \beta y v_g \\ \frac{dv_g}{dt} &= -fu_a - \beta y u_g \end{aligned} \quad (2)$$

Define  $\beta$ , and state in a list what assumptions you have made to get here.

**2a)** Over Boulder (40N), the 500 hPa relative vorticity is seen to increase by  $3 \times 10^{-6} \text{ s}^{-1}$  per hour. The wind is from the southwest at 20 m/s, and the relative vorticity decreases toward the northeast by  $4 \times 10^{-6} \text{ s}^{-1}$  over 100 km. It is unclear what to do this weekend. Start with equations (2) above to derive the quasi-geostrophic vorticity equation, write down an expression for divergence. Evaluate it for this case over Boulder.

**2b)** Turn your result into a forecast (KBCO called) - how might you describe the developing weather? Is this better news for Boulder's skiers or triathletes? (Explain your reasoning)

**3)** Show that in the quasi-geostrophic system, the quantity  $q$  is conserved during advection, if it is defined as:

$$q \equiv \frac{1}{f_0} \nabla^2 \Phi + f + \frac{\partial}{\partial p} \left( \frac{f_0}{\sigma} \frac{\partial \Phi}{\partial p} \right)$$

4) One strength of the quasi-geostrophic system is that all terms required to estimate the evolution of midlatitude weather systems can be computed knowing only the geopotential distribution. Assume that, on a  $\beta$ -plane, the geopotential describes a zonal jet upon which a transient zonal wave is superimposed:

$$\Phi = \Phi_0(p) + cf_0 \left\{ -y \left[ \cos\left(\frac{\pi p}{p_0}\right) + 1 \right] + \frac{1}{k} \sin k(x - ct) \right\} \quad (3)$$

$c$  is a phase speed of the wave, and  $k$  is the zonal wave number.  $\Phi_0$  is a function of pressure  $p$ , as given by the hydrostatic relation,

$$\Phi_0(p) = -gH \ln \frac{p}{p_0}$$

$p_0$  is a reference (surface) pressure of 1000 hPa, and  $f_0$  assumes typical midlatitude value, and for simplicity take  $\beta = 0$ .

Confirm that all quantities can be written in terms of the geopotential. Using (3):

**a** derive expressions for  $u_g$  and  $v_g$

**b** derive an expression for the  $\zeta_g$

**c** derive an expression for vorticity advection

**d** derive an expression for the quasi-geostrophic divergence

**e** derive an expression for the vertical velocity,  $\omega$ .

(assume the boundary condition that the vertical velocity vanishes at the surface)

5) Given your results for (4a-e) above, sketch (in the  $x$ - $p$  plane) the geopotential distribution at two pressure levels for a single wave length disturbance. Indicate locations of positive and negative vorticity, and positive and negative vorticity advection (with pluses and minuses). Also indicate divergence with diverging horizontal arrows (i.e., the approximate ageostrophic flow), and show vertical motion as up and down arrows.

This carefully about how to show all of this information *CLEARLY*. Color might help.