

## ATOC 5060: Atmospheric dynamics

### Homework exercise 2

Solution in class Tuesday

1. Consider a cylindrical tank with a flat bottom that is partially filled with water that rotates around its central axis (Holton's Figure 4.11). Derive an expression for the height of the surface (as a function of the radial distance from the rotation axis) of this shallow water layer as fluid rotates in solid body rotation with the tank for given rotation rate ( $\Omega$ ). Assume the depth in the center is known to be  $H$ . (*Hint: consider the forces involved*)
2. Consider Rossby's potential vorticity, which is an approximation of Ertel's potential vorticity. Under what conditions are the two equivalent in the atmosphere? (i.e., figure out what Ertel's PV is, then derive an expression that shows this equivalence)
3. Conservation of potential vorticity is often a key to simplifying many dynamical problems to their essence. Consider the rotating tank from above. Assuming the water is isothermal, how much would the (relative) vorticity of a column change if it is moved 0.5 m from the center toward the perimeter? The tank rotates at 20 revolution per minute, and the depth in the center is  $H = 0.1$  m. (Sign is important!)

Reading assignment (and good review for next week!): Have a browse<sup>1</sup> through Charney's 1948 paper that outlines scale analysis (PDF on web class site). Why does he call it potential vorticity? What is the physical interpretation?

Charney, J. G., On the scale of atmospheric motions, *Geogysike Publikasjoner*, 17(2), 1-17, 1948.

<sup>1</sup>The paper is quite long, and he goes through a number of arguments, and ultimately rejects some of them. This, plus his use of a different notation to us, makes it quite a difficult paper to follow. So "browse" the paper rather than study it in detail.