

Coriolis parameter

$$f = 2\Omega \sin \phi$$

Ideal gas law

$$p = \rho RT$$

Potential temperature

$$\theta = T \left(\frac{p}{p_0} \right)^{-R/c_p}$$

Buoyancy frequency

$$N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z}$$

Atmospheric angular momentum

$$M = (\Omega R + u)R$$

Phase speed of a wave

$$c_x = \frac{\nu}{k} \quad c_y = \frac{\nu}{l}$$

Group velocity of a wave

$$c_{gx} = \frac{\partial \nu}{\partial k} \quad c_{gy} = \frac{\partial \nu}{\partial l}$$

Stoke's theorem

$$\int_S (\nabla \times \vec{V}) \cdot d\vec{A} = \int_C \vec{V} \cdot d\vec{l}$$

Primitive equations (filtered)

$$\frac{du}{dt} = fv - \frac{\partial \Phi}{\partial x}$$

$$\frac{dv}{dt} = -fu - \frac{\partial \Phi}{\partial y}$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

$$\frac{dT}{dt} - \omega S_p = \frac{J}{c_p}$$

$$S_p = -\frac{T}{\theta} \frac{\partial \theta}{\partial p}$$

Shallow water equations

$$\frac{du}{dt} = fv - g \frac{\partial h}{\partial x}$$

$$\frac{dv}{dt} = -fu - g \frac{\partial h}{\partial y}$$

$$\frac{dh}{dt} = -H_0 \left(\frac{du}{dx} + \frac{dv}{dy} \right)$$

Quasi-geostrophic equations

$$\frac{du_g}{dt} = fv_a + \beta y v_g$$

$$\frac{dv_g}{dt} = -fu_a - \beta y u_g$$

$$\frac{\partial p}{\partial z} = -\rho g$$

$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

$$\frac{d}{dt} \left(-\frac{\partial \Phi}{\partial p} \right) - \omega \sigma = \frac{R}{p} \frac{J}{c_p}$$

$$\sigma = -\frac{RT_0}{p\theta} \frac{\partial \theta}{\partial p}$$

$$\frac{d\zeta_g}{dt} = -f_0 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - \beta v_g$$

Isentropic (Ertel) potential vorticity

$$Q = -g(\zeta + f) \left(\frac{\partial \theta}{\partial p} \right)$$

Rossby potential vorticity

$$P = \left(\frac{\zeta + f}{h} \right)$$

Quasi-geostrophic potential vorticity

$$q \equiv \frac{1}{f_0} \nabla^2 \Phi + f + \frac{\partial}{\partial p} \left(\frac{f_0}{\sigma} \frac{\partial \Phi}{\partial p} \right)$$

Constants

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|--------------------------------------|--|
| Radius of Earth | $a = 6.370 \times 10^6 \text{ m}$ |
| Planetary rotation rate | $\Omega = 7.292 \times 10^{-5} \text{ rad s}^{-1}$ |
| Acceleration due to gravity | $g = 9.81 \text{ ms}^{-2}$ |
| Gas constant for dry air | $R = 287 \text{ J K}^{-1} \text{ kg}^{-1}$ |
| Gas constant for water vapor | $R_v = 461 \text{ J K}^{-1} \text{ kg}^{-1}$ |
| Heat capacity (at constant pressure) | $c_p = 1005 \text{ J K}^{-1} \text{ kg}^{-1}$ |
| Latent heat of condensation | $L = 2.5 \times 10^6 \text{ J kg}^{-1}$ |
| Scale height (troposphere) | $H = 7600 \text{ m}$ |
| Dry adiabatic lapse rate | $\Gamma_d = 9.8 \text{ K km}^{-1}$ |
| Moist adiabatic lapse rate | $\Gamma_m = 6.5 \text{ K km}^{-1}$ |
| Typical Coriolis parameter | $f_0 \sim 1 \times 10^{-4} \text{ s}^{-1}$ |
| Typical beta parameter | $\beta \sim 1 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$ |