

## Diagnosis of vertical motion

### Geopotential tendency

QG vorticity equation: 
$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \cdot \nabla (\zeta_g + f) + f_0 \frac{\partial \omega}{\partial p}$$

QG thermodynamic equation: 
$$\left( \frac{\partial}{\partial t} + \mathbf{V}_g \cdot \nabla \right) \left( -\frac{\partial \Phi}{\partial p} \right) - \sigma \omega = \frac{R}{c_p} \frac{J}{p}$$

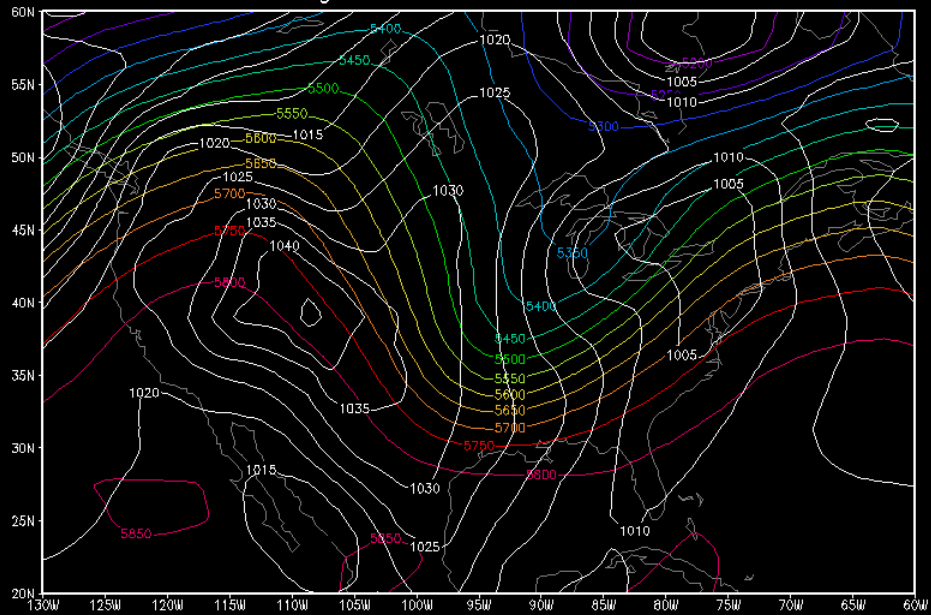
- As a *consequence of conservation of QG PV*

$$\left[ \nabla^2 + \frac{\partial}{\partial p} \left( \frac{f_0^2}{\sigma} \frac{\partial}{\partial p} \right) \right] \frac{\partial \Phi}{\partial t} = -f_0 \mathbf{V}_g \cdot \nabla \left( \frac{1}{f_0} \nabla^2 \Phi + f \right) - \frac{\partial}{\partial p} \left[ \frac{f_0^2}{\sigma} \mathbf{V}_g \cdot \nabla \frac{\partial \Phi}{\partial p} \right]$$

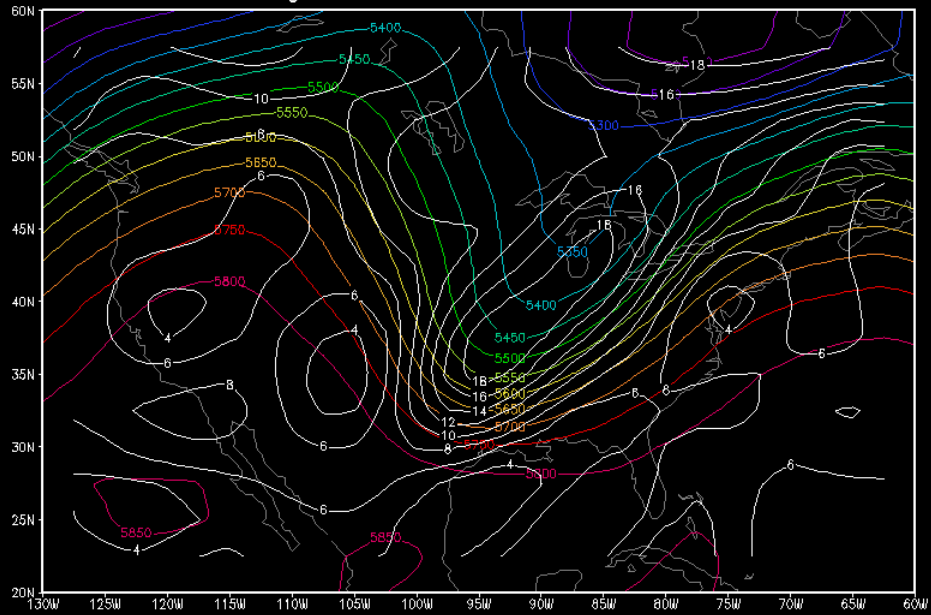
Notice left hand side is an elliptic operator for the tendency (which can be solved given boundary conditions)

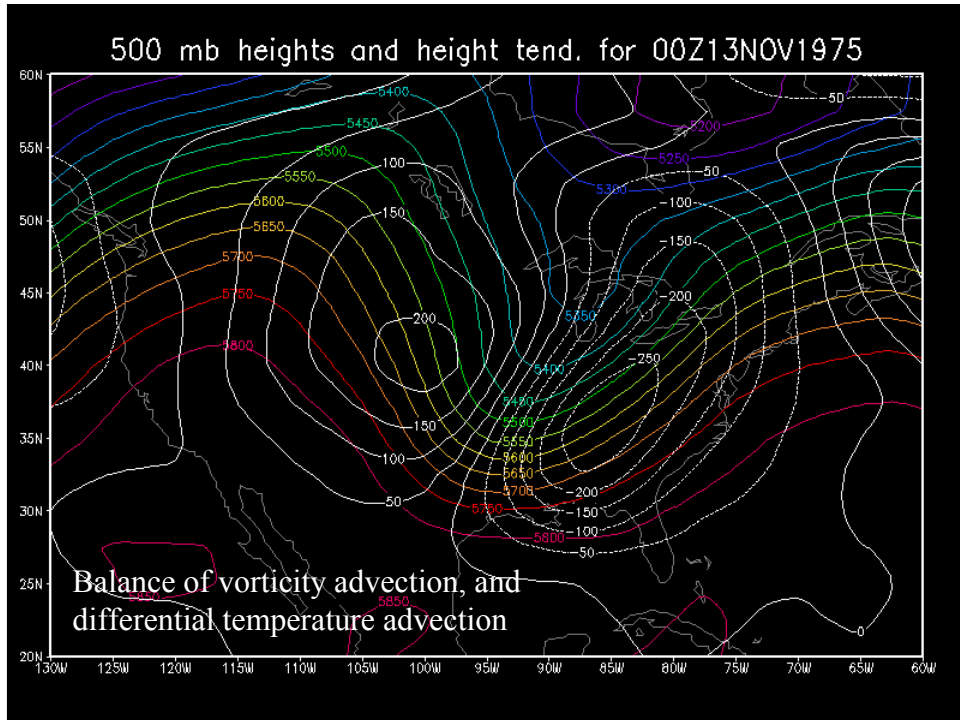
Terms on the right are all in terms of geopotential

### 500 mb heights and MSLP for 00Z13NOV1975



### 500 mb heights and abs. vort. for 00Z13NOV1975





## Vertical motion

- In deriving geopotential equation we canceled “omega”.
- Could combine to cancel geopotential tendency, to get omega as a function of geopotential.

## QG vertical motion

- $\omega$  was eliminated in prediction equation, but is important for, for instance, stretching.
- Specifically, if we want the ageostrophic divergence, need to be able to extract it from horizontal geostrophic flow – recall divergence appear in vorticity equation
- Consider momentum equation? Continuity? Thermodynamic? Each  $\omega$  as a residual from a difference in large quantities.

$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \quad \left( \frac{\partial}{\partial t} + \mathbf{V}_g \cdot \nabla \right) \left( -\frac{\partial \Phi}{\partial p} \right) - \sigma \omega = \frac{R}{c_p} \frac{J}{p}$$

- Expect most accurate method to be based on *state* of geopotential rather than *change* in geopotential (as, e.g., in vorticity equation)

## Omega equation

- Merge QG vorticity and thermodynamic equations
- To derive the geopotential tendency, combined to eliminate  $\omega$
- Now, eliminate tendency terms, and retain  $\omega$

$$\left( \nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega = \frac{f_0}{\sigma} \frac{\partial}{\partial p} \left[ \mathbf{V}_g \cdot \nabla \left( \frac{1}{f_0} \nabla^2 \Phi + f \right) \right] + \frac{1}{\sigma} \nabla^2 \left[ \mathbf{V}_g \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) \right] - \frac{\kappa}{\sigma p} \nabla^2 J$$

vorticity	temperature	diabatic
advection	advection	heating

For adiabatic conditions, scale analysis gives diagnostic equation:

$$\left( \nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega \approx \frac{f_0}{\sigma} \left[ \frac{\partial \mathbf{V}_g}{\partial p} \cdot \nabla \left( \frac{1}{f_0} \nabla^2 \Phi + f \right) \right]$$

*Depends only on geopotential!*

*Not divergence! Not vorticity tendency! Not temperature tendency!*

## Recall thermal wind

$$w \propto \left( \nabla^2 + \frac{f_0^2}{\sigma} \frac{\partial^2}{\partial p^2} \right) \omega \approx \frac{f_0}{\sigma} \left[ \frac{\partial \mathbf{V}_g}{\partial p} \cdot \nabla \left( \frac{1}{f_0} \nabla^2 \Phi + f \right) \right]$$

*i.e., since “Laplacian” give opposite sign*

$$\mathbf{V}_T = \frac{\partial \mathbf{V}_g}{\partial p} = \frac{R}{f} k \times \nabla T \ln \left( \frac{p_0}{p} \right)$$

So vertical motion associated with “advection of absolute vorticity by the thermal wind”

$$w \propto \mathbf{V}_T \cdot \nabla (\zeta + f)$$

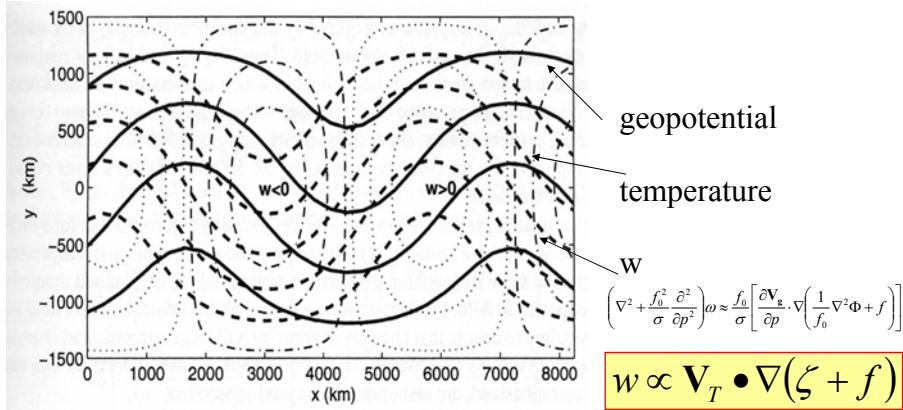
Thermal wind parallel to isotherms (e.g., 500-1000 hPa thickness)

*This has immediate use in understanding weather!*

## Properties of omega

- Obtain accurate  $\omega$  diagnostically (recall no acceleration in hydrostatic equation)
- Ascent ( $\omega < 0$ ,  $w > 0$ )
- Descent ( $\omega > 0$ ,  $w < 0$ )
  
- Vertical velocity exists to ensure thermal wind balance in the presence of different vorticity advection at different levels
  
- *All good!*

# QG vertical motion



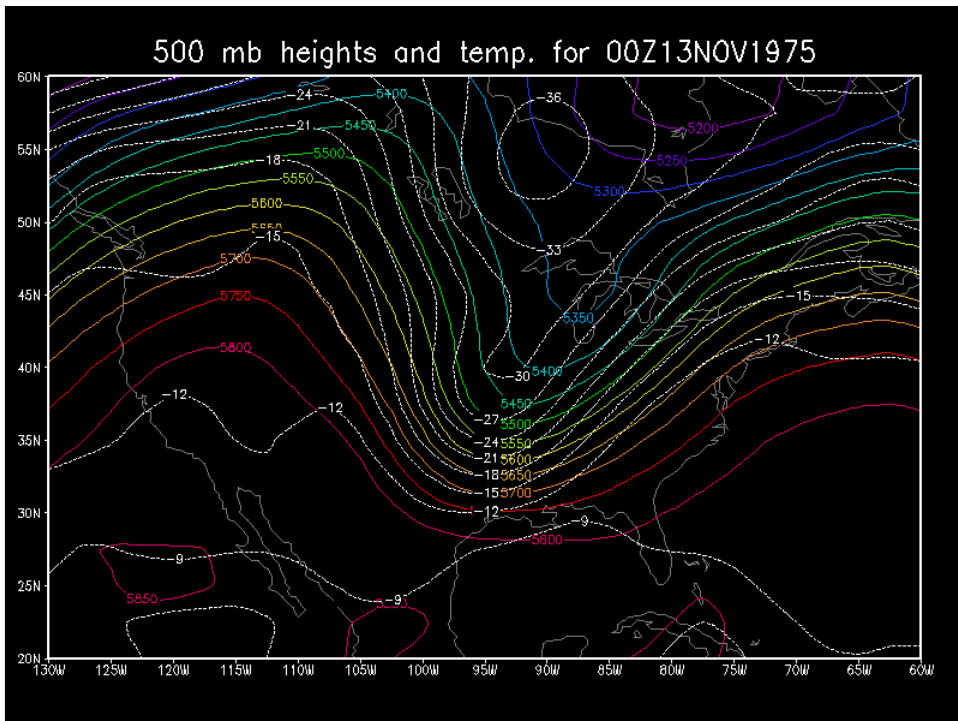
Offset isobars and isotherms due to westward tilt

Cold temperature advection implicates thermal wind

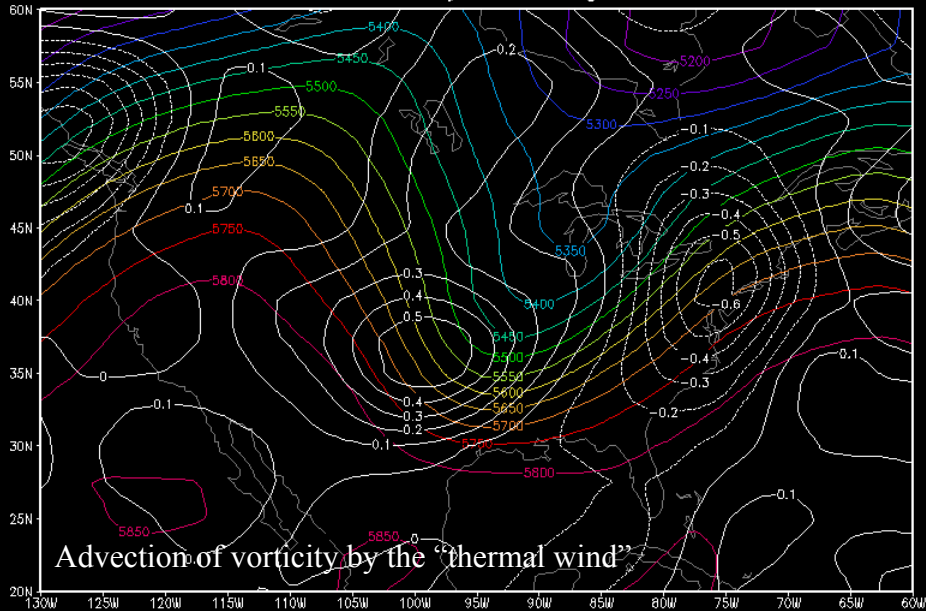
So vertical velocity must exist for balance

$\omega$  qualitatively by looking at vorticity advection ALONG isotherms

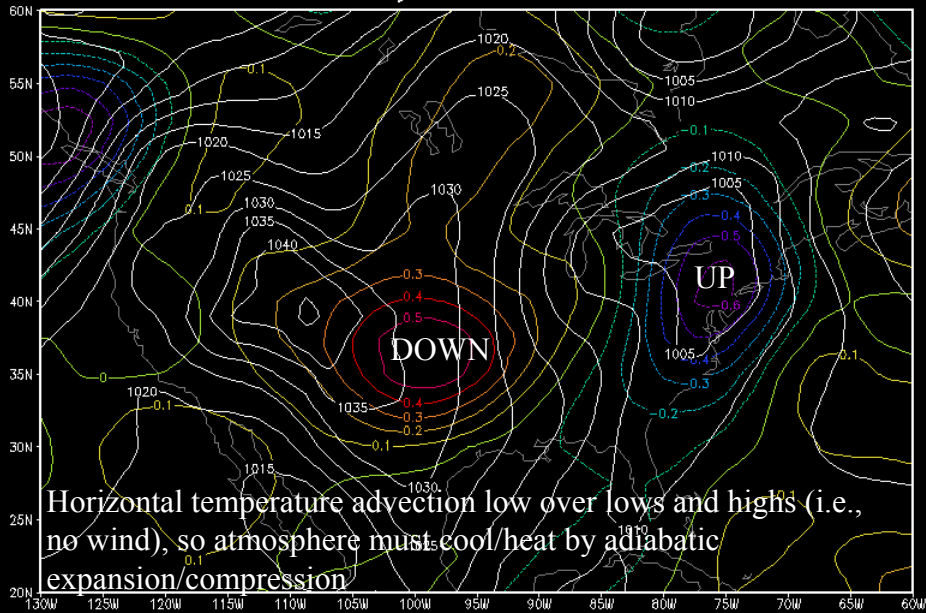
Vorticity advection at maximum above surface low, thus ascent



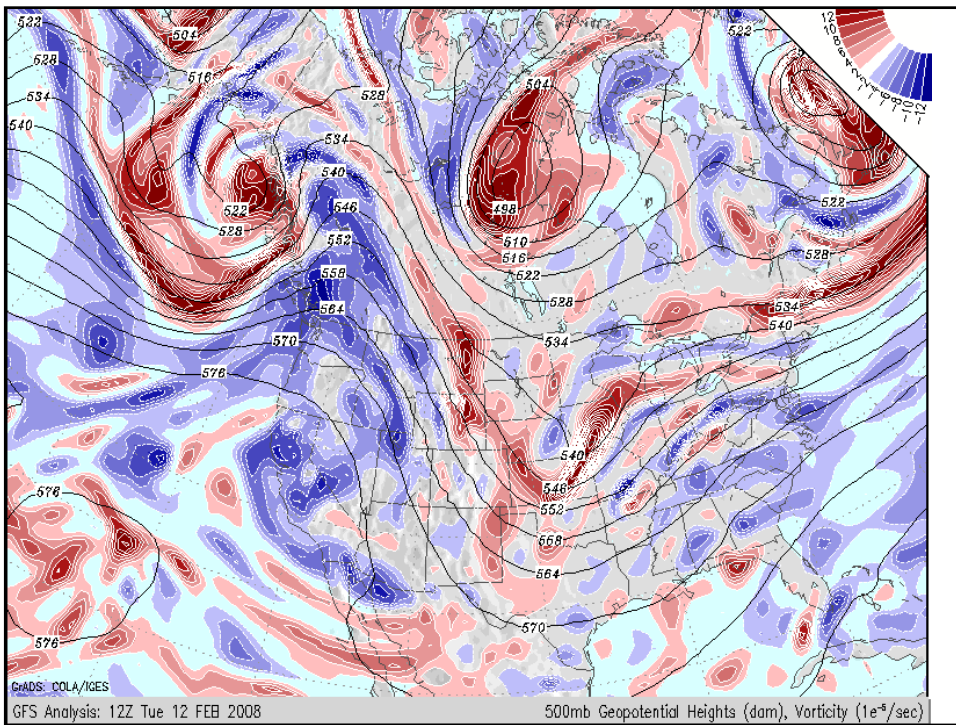
500 mb 500 mb vert. velocity and heights for 00Z13NOV1975

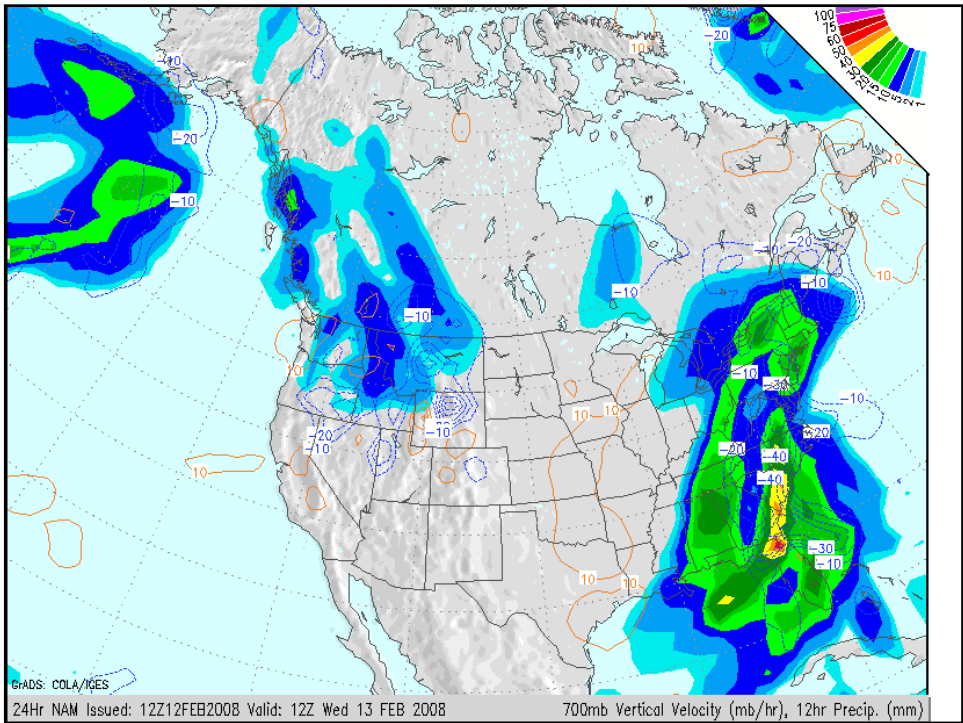
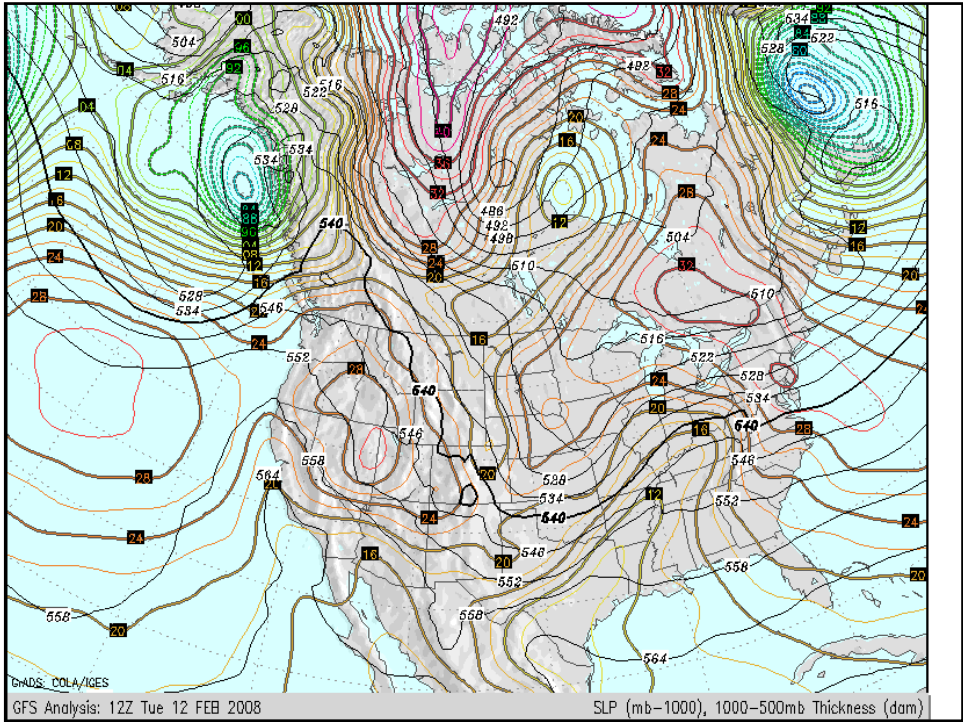


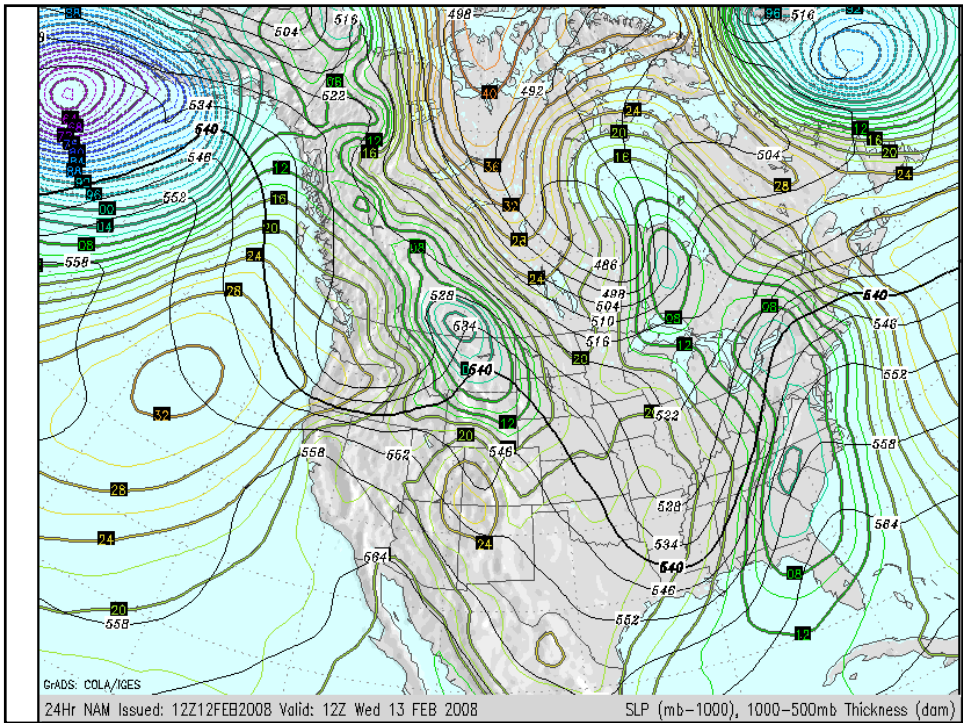
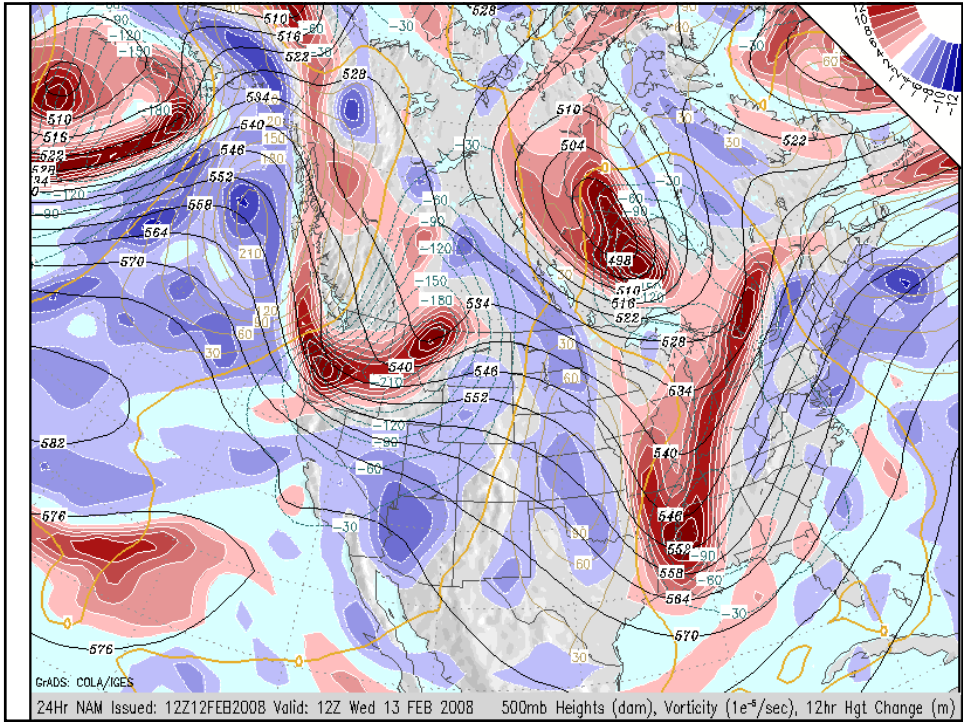
500 mb vert. velocity and MSLP for 00Z13NOV1975

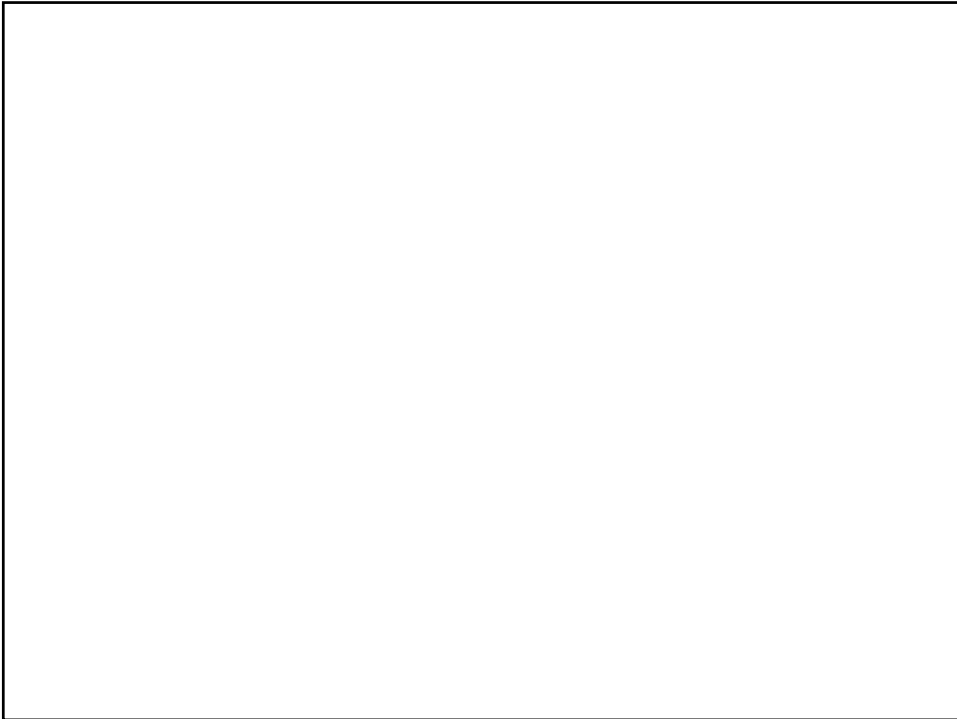


Today...?









## QG wrap up

- What are the key assumptions?
- What is the role of divergence?
- Why is QG PV useful (what are its special properties, and where does it come from)?
- What are the relationships between the various variables of interest?  
(mathematical, sure, but what is the physical meaning)?
- Why was (is!) QG useful for weather forecasting?
  
- QG can allow new weather features develop.
- To understand this in more detail, it is useful to consider how atmospheric waves can amplify.