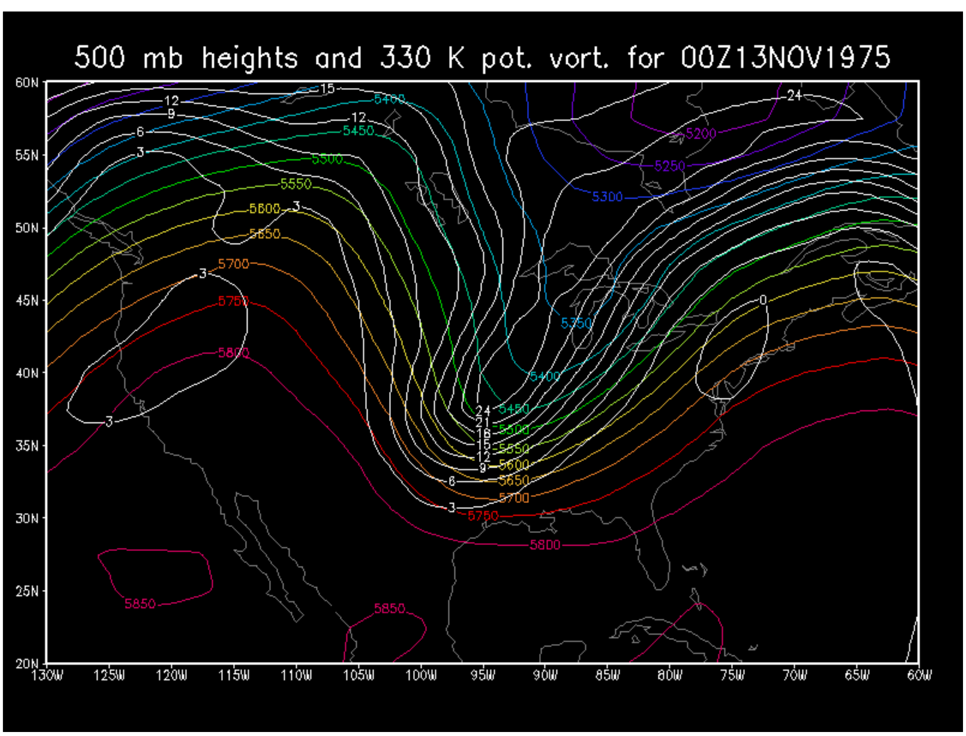
Geopotential tendency and vertical motion



Recall PV inversion…

- Knowing the PV, we can estimate everything else! (Temperature, wind, geopotential …)
- In QG, since the flow is geostrophic, we can obtain the wind field exactly. Then with thermal wind balance, obtain the temperature exactly.
- More generally (i.e., not QG), need to assume a number of "balance" conditions to do the PV inversion.

Example: PV inversion with ozone

•Ozone concentrated in stratosphere

•PV concentrated in stratosphere (stronger stability)

•PV conservation means PV behaves like a passive tracer (much like ozone)

So, could use measurements of ozone to guess PV field

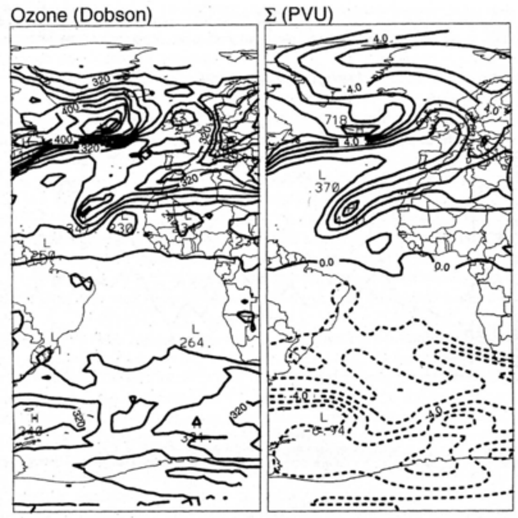
then invert this derived PV to obtain wind field

Then use thermal wind equation to deduce temperature field

Would be useful near tropopause and lower stratosphere where wind measurements are difficult

Would this work?

Ozone measured from TOMS



PV calculated from weather observations

Figure 1. Integrated ozone (left; Dobson Units, DU), and Ertel potential vorticity (PV) integrated between 500 mb and 50 mb (right; normalized to be in PVU (see text)). Contour interval on the left is 20 DU; on the right it is 1 PVU, with negative PV denoted by dashed lines. Geographic region is sector 3, the Atlantic sector. Time is 1200 UTC 17 February 1997.

Davis et al. (1999), QJRMS, **125**, 3375-3391

200hPa wind from PV_{ozone}

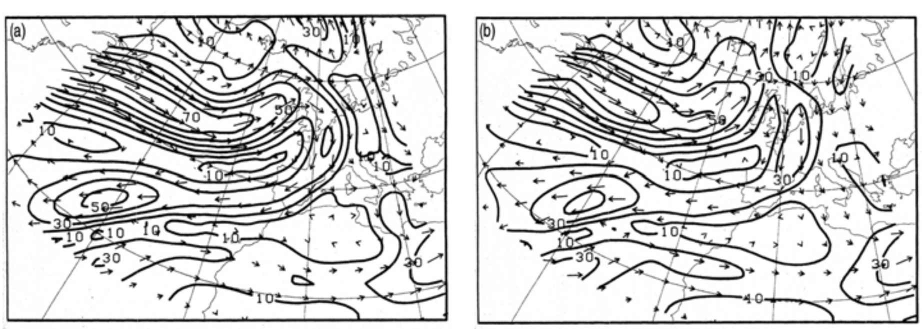


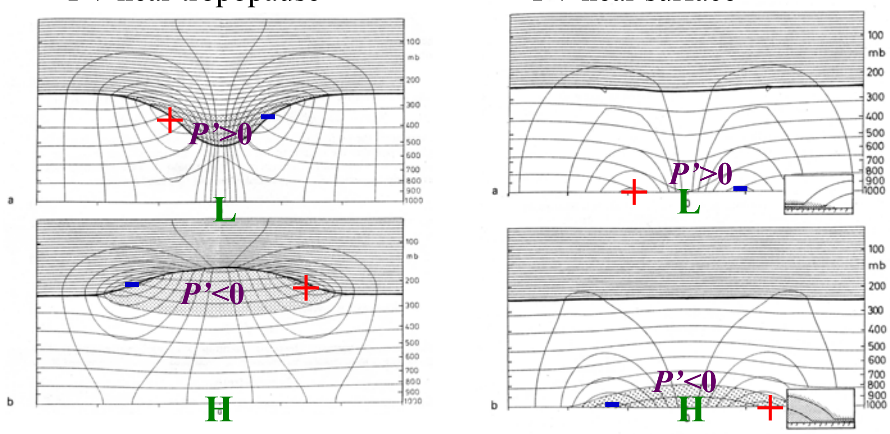
Figure 7. Balanced, nondivergent wind at 250 mb obtained from inverting (a) analysed potential vorticity (PV) and (b) ozone-derived PV. Contour interval for velocities is 10 m s^{-1}. Vectors are displayed at alternate grid points. A single smoothing pass has been applied to the contours.

Davis et al. (1999), QJRMS, **125**, 3375-3391

Example: more PV anomalies

PV near tropopause

PV near surface



PV anomaly shaded
Potential temperature, and wind anomalies contoured

Hoskins et al. (1985), QJRMS, **111**, 877-946

Example: Vertical coupling forced at 250 hPa

Amount of vertical inversely proportional to length scale
(as Laplacian "selects" for smaller scale).

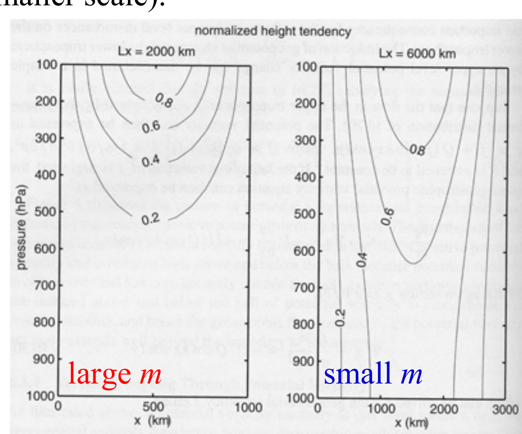
Consider scale (via m^{-1}):

$$\psi = \sin mz$$

$$\frac{\partial \psi}{\partial z} = m \cos mz$$

$$\frac{\partial^2 \psi}{\partial z^2} = -m^2 \sin mz$$

$$= -m^2 \psi$$



Again, an artifact of the "smoothing" associated with the eleptic

Example: Impact of aspect ratio

Recall vertical and horizontal scales are related to f and σ. Specifically, $k/m \sim f^2/\sigma$

Consider scale (via k^{-1}):

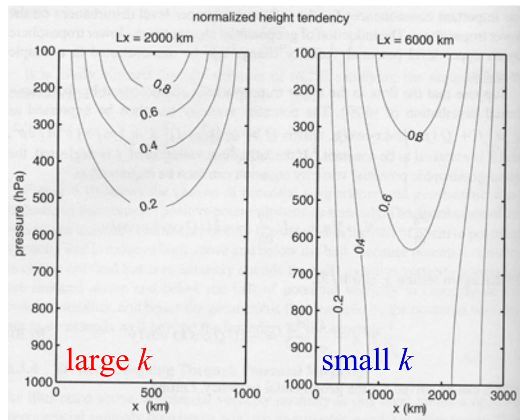
$$\psi = \sin mz \sim \sin kx$$

$$\frac{\partial \psi}{\partial z} \sim k \cos kx$$

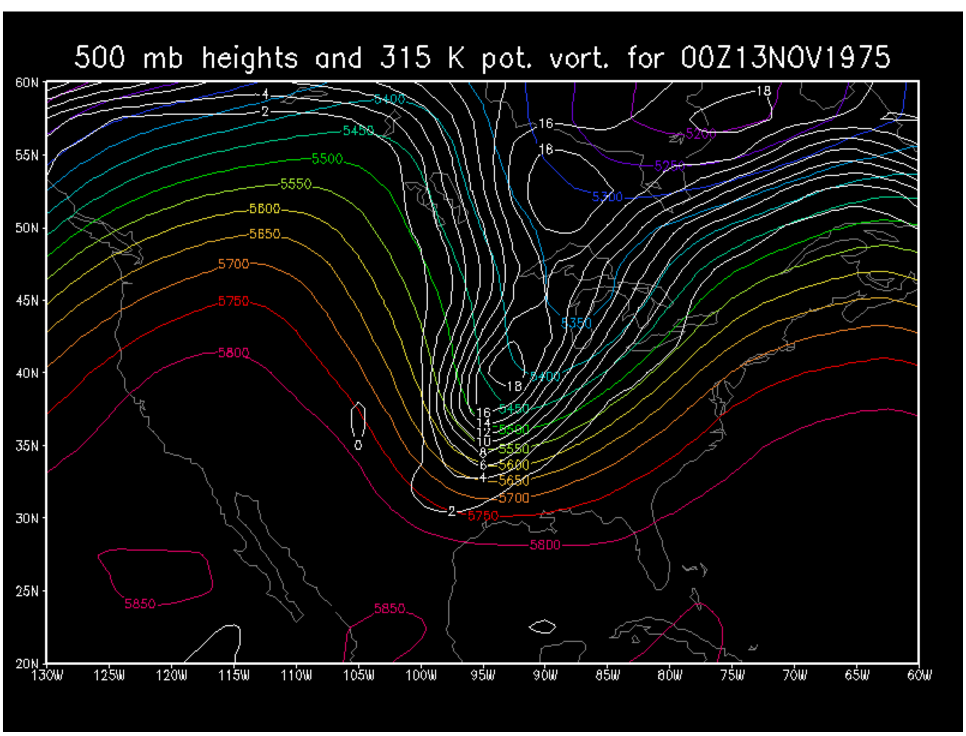
$$\frac{\partial^2 \psi}{\partial z^2} \sim -k^2 \sin kx$$

$$\sim -k^2 \psi$$

So more vertical propagation for longer waves (k small, k^{-1} large)



In future we will see that this means, for long waves, upper troposphere flow, can mess with lower troposphere flow.



Geopotential tendency

(a synoptic meteorology view of QG)

i.e., "PV is nice and all, but geopotential makes more sense to me!"

Prognostic geopotential equation

QG vorticity equation:

$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \cdot \nabla(\zeta_g + f) + f_0 \frac{\partial \omega}{\partial p}$$

QG thermodynamic equation:

$$\left(\frac{\partial}{\partial t} + \mathbf{V_g} \cdot \nabla\right)\left(-\frac{\partial \Phi}{\partial p}\right) - \sigma\omega = \frac{R}{c_p}\frac{J}{p}$$

- As a ***consequence of conservation of QG PV***

$$\left[\nabla^2 + \frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\frac{\partial \Phi}{\partial t} = -f_0 \mathbf{V}_g \cdot \nabla\left(\frac{1}{f_0}\nabla^2\Phi + f\right) - \frac{\partial}{\partial p}\left[\frac{f_0^2}{\sigma}\mathbf{V}_g \cdot \nabla \frac{\partial \Phi}{\partial p}\right]$$

Notice left hand side is an elliptic operator for the tendency (which can be solved given boundary conditions)

Terms on the right are all in terms of geopotential

Geopotential tendency equation

$$\left[\nabla^2+\frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\frac{\partial\Phi}{\partial t}=-f_0\mathbf{V}_g\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi+f\right)-\frac{\partial}{\partial p}\left[\frac{f_0^2}{\sigma}\mathbf{V}_g\cdot\nabla\frac{\partial\Phi}{\partial p}\right]$$

vorticity advection

temperature advection

- Thermal advection dominates development.
  (stronger in lower troposphere)
- Through thermal wind balance, ensures that low level development is associated with upper level change (surface cyclogenesis causes upper level trough, etc)
- Notice this suggests a transfer of energy from potential energy (related to thickness/goepotential height) to kinetic energy (more vorticity, and stronger wind speeds)
- This conversion is fundamental to the general circulation

QG prediction

QG vorticity $$\frac{\partial\zeta_g}{\partial t}=-\mathbf{V}_g\cdot\nabla(\zeta_g+f)+f_0\frac{\partial w}{\partial p}$$

vorticity advection

vorticity stretching (ageostrophic)

$$\zeta_g=\frac{1}{f_0}\nabla^2\Phi$$

Given Φ, can compute u_g and v_g.
What about vertical velocity?

Eliminate it! (Use thermodynamic equation.)

$$\left[\nabla^2+\frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\frac{\partial\Phi}{\partial t}=-f_0\mathbf{V}_g\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi+f\right)-\frac{\partial}{\partial p}\left[\frac{f_0^2}{\sigma}\mathbf{V}_g\cdot\nabla\frac{\partial\Phi}{\partial p}\right]$$

vorticity advection

Vertical difference in temperature advection

Recall Laplacian gives $d\Phi < 0$ when $d\zeta > 0$, etc

Inverting Laplacian tends to smooth. (And give a minus sign)
Also, elliptic equations have solutions!

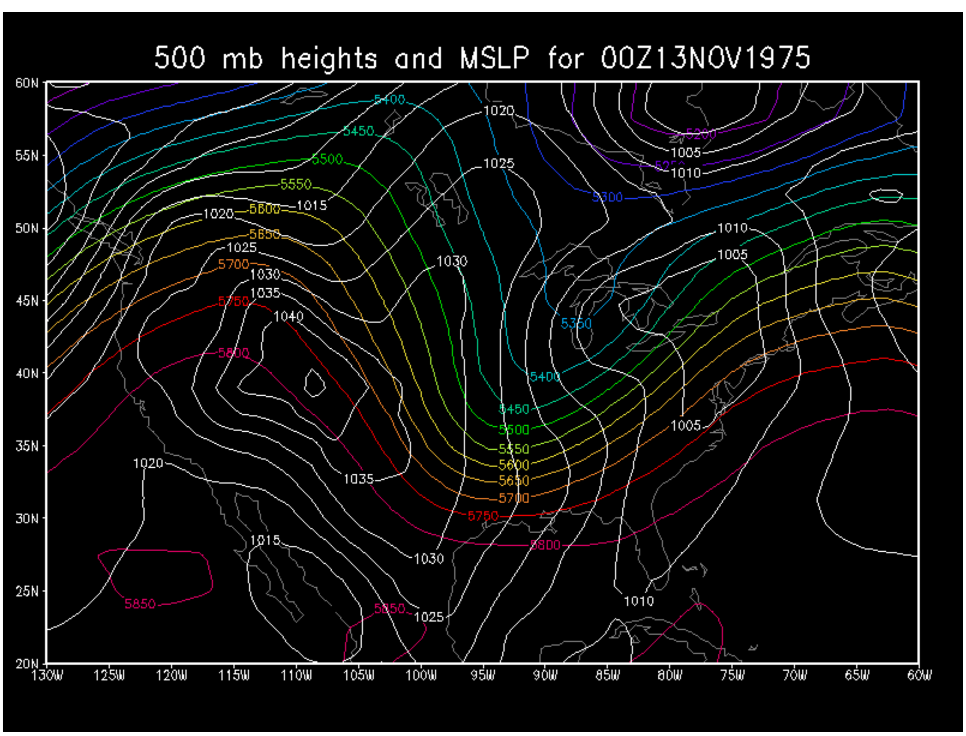
Geopotential tendency

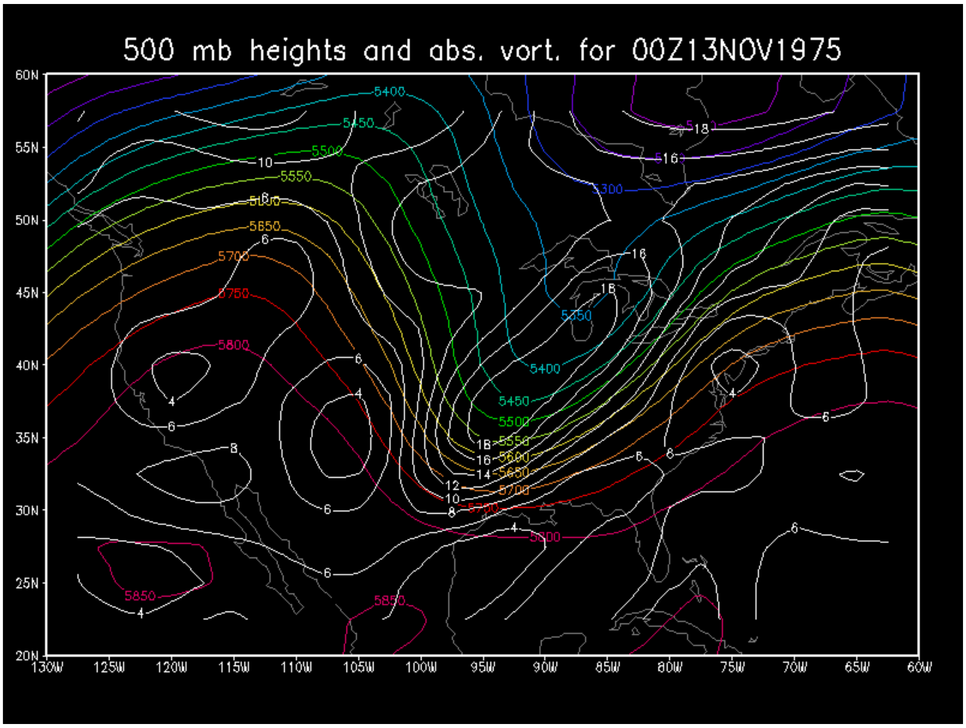
$$\left[\nabla^2+\frac{\partial}{\partial p}\left(\frac{f_0^2}{\sigma}\frac{\partial}{\partial p}\right)\right]\frac{\partial\Phi}{\partial t}=-f_0\mathbf{V}_g\cdot\nabla\left(\frac{1}{f_0}\nabla^2\Phi+f\right)-\frac{\partial}{\partial p}\left[\frac{f_0^2}{\sigma}\mathbf{V}_g\cdot\nabla\frac{\partial\Phi}{\partial p}\right]$$

- Φ falls with (positive) vorticity advection (cyclonic)
- Φ falls with when warm air advection increases with height (or when cold air advection decreases with height)

Example:

- Warm air advection at the surface causes increased thickness
- Increased thickness causes high pressure at layer top
- High pressure creates ageostrophic horizontal motion
- Unbalanced ageostrophic drives divergence (mass lost from layer)
- Divergence lowers heights, and creates low pressure at surface (still high pressure at layer top)
- Surface low causes convergent ageostrophic motion for balance
- ***So knowing geopotential can estimate "omega" with QG!***





QG vertical motion

- ω was eliminated in prediction equation, but is important for, for instance, stretching.
- Specifically, if we want the ageostrophic divergence, need to be able to extract it from horizontal geostrophic flow
- Different estimates of ω are subject to different errors in equations (this is a practical issue)
- Consider momentum equation? Continuity? Thermodynamic? Each ω as a residual from a difference in large quantities.

$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial \omega}{\partial p} = 0 \qquad \left(\frac{\partial}{\partial t} + \mathbf{V_g} \cdot \nabla\right)\left(-\frac{\partial \Phi}{\partial p}\right) - \sigma\omega = \frac{R}{c_p}\frac{J}{p}$$

- Expect most accurate method to be based on *state* of geopotential rather than *change* in geopotential
  (as, e.g., in vorticity equation)