Quasi-geostrophic system

(or, *why we love elliptic equations for QGPV*)

Charney's QG

- *"the motion of large-scale atmospheric disturbances is governed by…*
- *Laws of conservation of potential temperature*
- *and potential vorticity,*
- *and by the conditions that the horizontal velocity be quasi-geostrophic*
- *and the pressure quasi-hydrostatic"*

(Section 2, On the scale of atmospheric motions, 1948)

Baroclinic cyclogenesis

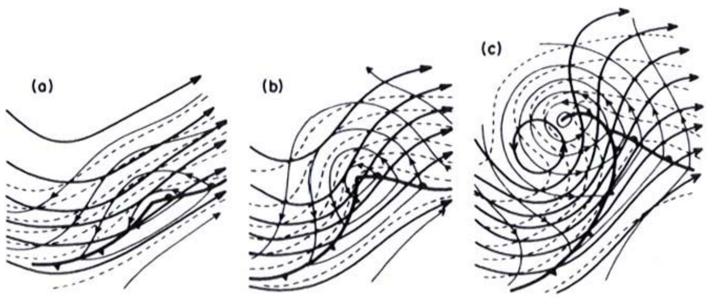
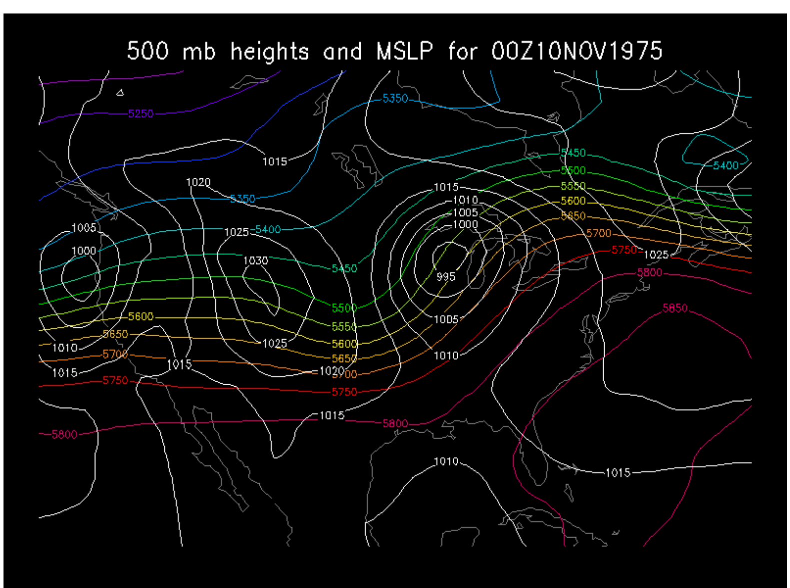
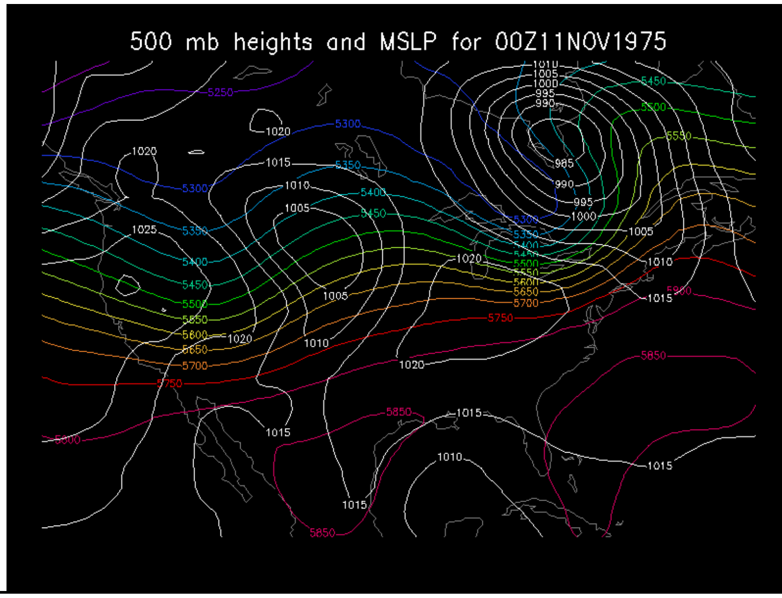


Fig. 6.5 Schematic 500-hPa contours (heavy solid lines), 1000-hPa contours (thin lines), and 1000–500 hPa thickness (dashed) for a developing baroclinic wave at three stages of development. (After Palmén and Newton, 1969.)

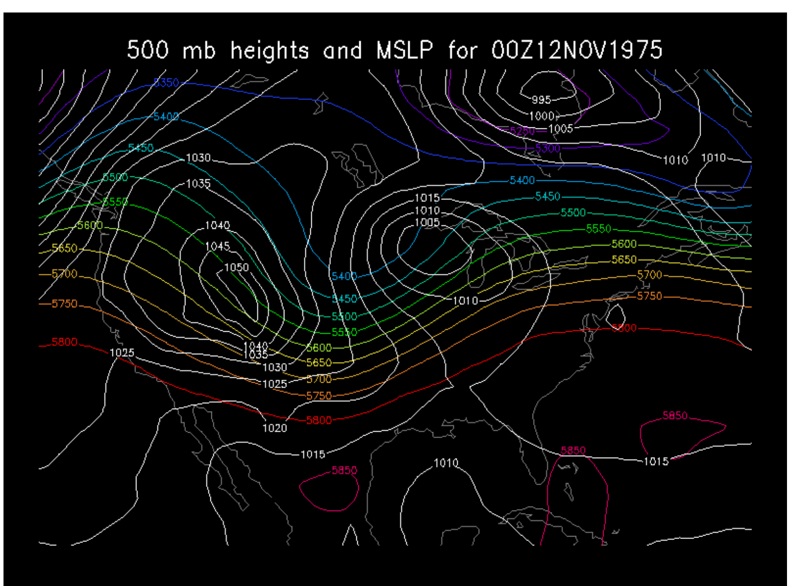
Cyclone development



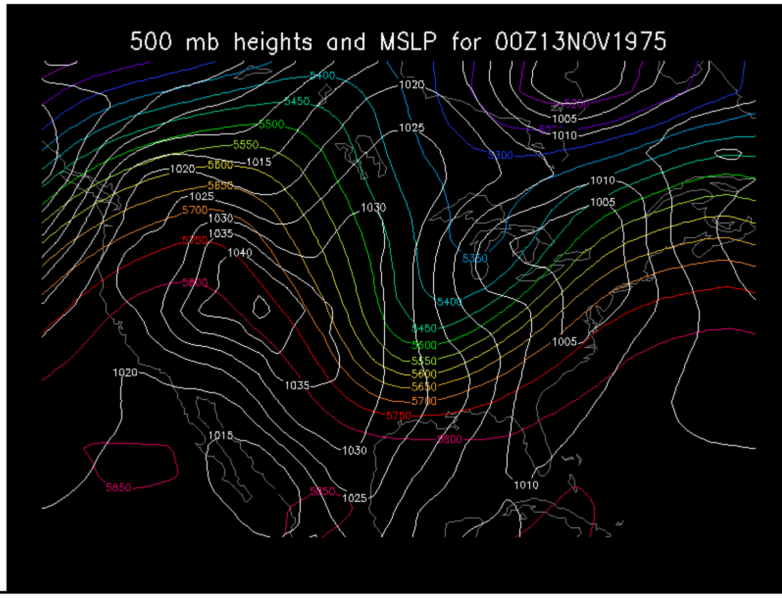
Cyclone development



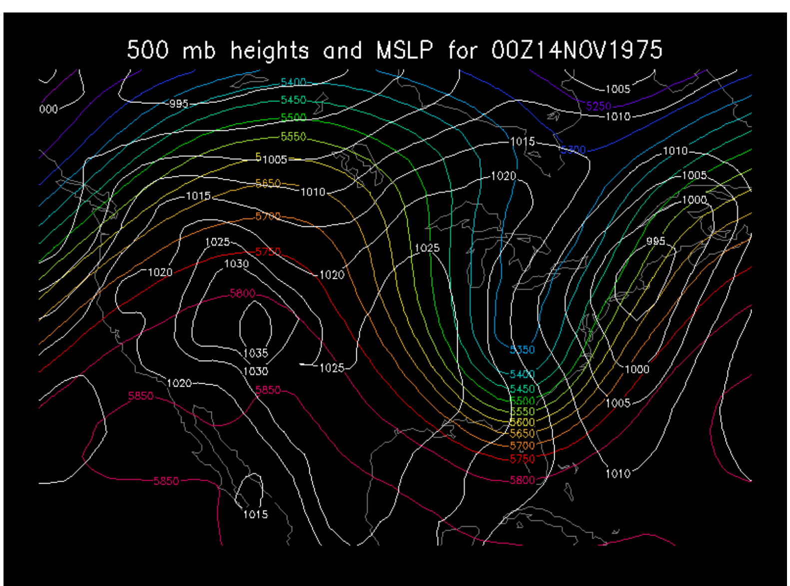
Cyclone development



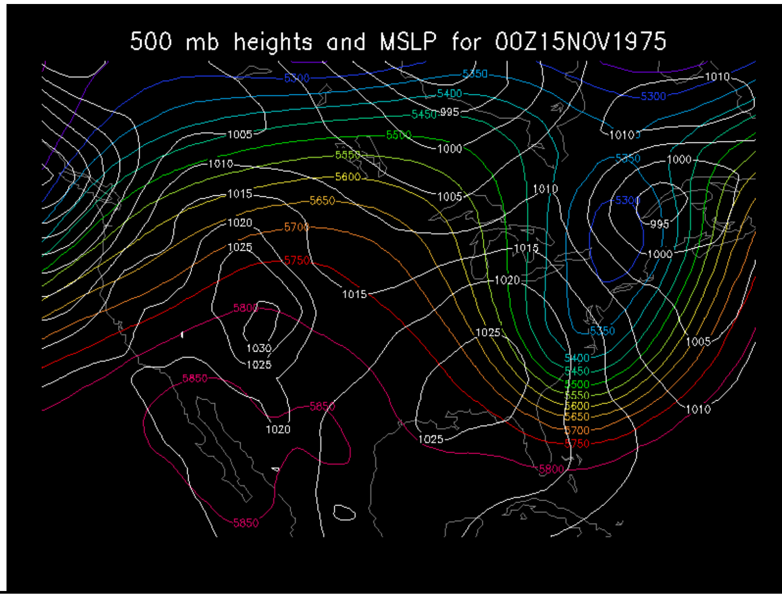
Cyclone development



Cyclone development



Cyclone development



Geostrophic vorticity

- Define quasi geostrophic vorticity,

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}$$

Geostrophic wind equations, give

$$v_g = \frac{1}{f_0}\frac{\partial \Phi}{\partial x} \qquad u_g = -\frac{1}{f_0}\frac{\partial \Phi}{\partial y}$$

$$\zeta_g = \frac{1}{f_0}\left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}\right) = \frac{1}{f_0}\nabla^2 \Phi$$

Since flow is non-divergent, we may define a streamfunction

$$\zeta_g = \nabla^2 \psi = \frac{1}{f}\nabla^2 \Phi$$

Knowing streamfunction (geopotential),
can invert for vorticity

QG vorticity equation

- Much like derivation of vorticity equation, take $\partial/\partial x$ of v_g equation minus $\partial/\partial y$ of u_g equation

$$\frac{\partial \zeta_g}{\partial t} = -\mathbf{V}_g \cdot \nabla\left(\zeta_g + f\right) + f_0 \frac{\partial w}{\partial p}$$

cf. primitive vorticity equation (scaled vorticity equation)

$$\frac{d\zeta}{dt} = -\mathbf{V} \cdot \nabla(\zeta + f) - \omega \frac{\partial \zeta}{\partial p} - (\zeta + f)\nabla \cdot \mathbf{V} + \mathbf{k} \cdot \left(\frac{\partial \mathbf{V}}{dp} \times \nabla \omega\right)$$

Recall vorticity equation lead us to the Rossby potential vorticity

Expect to find a QG potential vorticity in the not to distant future!

Steps in QG derivation

- Horizontal flow is geostrophic
  (horizontal advection by geostrophic flow)
- Coriolis effects beyond that needed for geostrophic balance due to
  1. Ageostrophic flow
  2. Variation in Coriolis parameter with latitude
- Vertical motion due to (ageostrophic) divergence

- Atmosphere is hydrostatic
- Atmosphere ("air") is an ideal gas
- Conservation of mass

Charney's QG

- *"the motion of large-scale atmospheric disturbances is governed by…*
- *Laws of conservation of potential temperature*
- *and potential vorticity,*
- *and by the conditions that the horizontal velocity be quasi-geostrophic*
- *and the pressure quasi-hydrostatic"*

(Section 2, On the scale of atmospheric motions, 1948)

Filtered equations

Primitive equations

($\mathbf{V}$, Φ, ω, T, given J)

$$\frac{d\mathbf{V}}{dt} = -f\mathbf{k}\times\mathbf{V} - \nabla\Phi$$

$$\frac{\partial\Phi}{\partial p} = -\frac{RT}{p}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)T - S_p\omega = \frac{J}{c_p}$$

5 independent quantities

Quasi-Geostrophic equations

($\mathbf{V_g}$, $\mathbf{V_a}$, Φ, ω, given J)

$$\frac{d\mathbf{V_g}}{dt} = -f_0\mathbf{k}\times\mathbf{V_a} - \beta y\mathbf{k}\times\mathbf{V_g}$$

$$\frac{\partial\Phi}{\partial p} = -\frac{RT}{p}$$

$$\frac{\partial u_a}{\partial x} + \frac{\partial v_a}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V_g}\cdot\nabla\right)\left(-\frac{\partial\Phi}{\partial p}\right) - \sigma\omega = \frac{R}{c_p}\frac{J}{p}$$

All variables related to Φ! (or q)

QG system

Advantages:

- changes in Z, T, and V are linked
- relations between variables are simple (expediting the math and the interpretation)
- energetically consistent
- captures the gross features observed in middle latitudes

Some properties:

- T (or θ) is proportional to $\partial\Phi/\partial p$
- advect**ed** velocities are geostrophic.
- advect**ing** velocities are geostrophic
- Vertical “advection” contribution in thermodynamic equation associated with adiabatic work (expansion/compression)
- $d\theta/dt = -\omega S$ where S is a horizontal mean static stability.
- vorticity equation has 3 parts:
  - local change
  - horizontal advection of geostrophic absolute vorticity (both relative and planetary)
  - -fD divergence term.

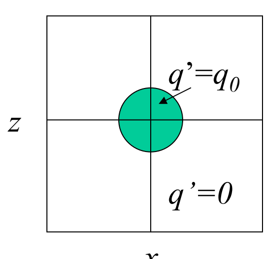
QG prediction

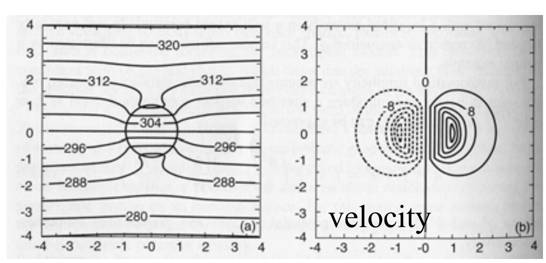
- A barotopic situation would dictate the advection of vorticity can be used to show estimate changes in vorticity, an thus geopotential height
- As we know thermal wind balance describes balance between horizontal flow at different levels.
- So, change in the vertical wind shear will induce vertical motion (stretching), and ageostrophic flow to maintain is ***required*** the thermal wind balance
- These changes can be larger than the advective role!
- However, it is the vertical motion induced by different vorticity advection at two different layer that is of central interest.
- Similarly, thermal advection will be associated with vertical motion, as the vorticity at one height will change relative to another.
- To close our system, we must remove the vertical velocity from the equations
(it appears in BOTH vorticity and thermodynamic equation)
- It is from here that the QG potential vorticity emerges

PV inversion

- PV related to geopotential by elliptic operator, since $\zeta_g = \frac{1}{f_0}\nabla^2\Phi$
- Consider PV field: $q = q' + f$

With a spherical anomaly $$q' = \frac{1}{f_0 r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi'}{\partial r}\right)' = \begin{cases} q_0 & \mathrm{r} \le \mathrm{b} \\ 0 & \mathrm{r} > \mathrm{b} \end{cases}$$





•Recall Laplacian tends to smooth
(as in weather maps, and Photoshop)

•Φ obtained from PV tends to "spread" beyond PV features

•This gives a type of "action at a distance"

Example: PV inversion with ozone

- Ozone concentrated in stratosphere
- PV concentrated in stratosphere (stronger stability)
- PV conservation means PV behaves like a passive tracer (much like ozone)

So, could use measurements of ozone to guess PV field

then invert this derived PV to obtain wind field

Then use thermal wind equation to deduce temperature field

Would be useful near tropopause and lower stratosphere where wind measurements are difficult

Would this work?

Ozone measured from TOMS

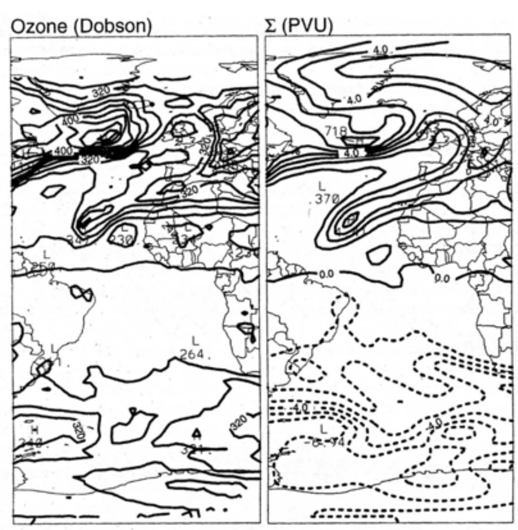


Figure 1. Integrated ozone (left; Dobson Units, DU), and Ertel potential vorticity (PV) integrated between 500 mb and 50 mb (right; normalized to be in PVU (see text)). Contour interval on the left is 20 DU; on the right it is 1 PVU, with negative PV denoted by dashed lines. Geographic region is sector 3, the Atlantic sector. Time is 1200 UTC 17 February 1997.

PV calculated from weather observations

Davis et al. (1999), QJRMS, **125**, 3375-3391

200hPa wind from PV_{ozone}

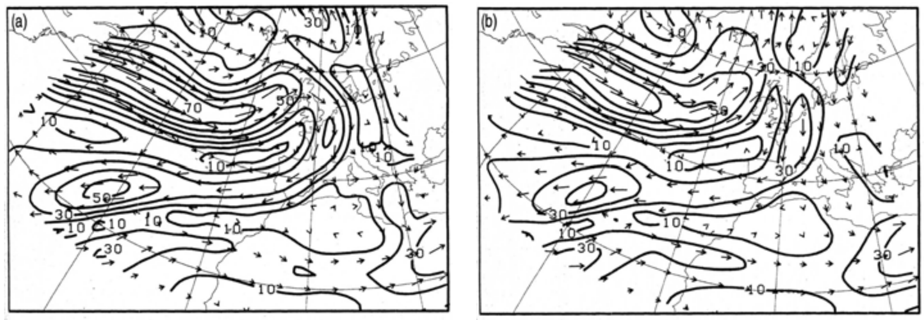


Figure 7. Balanced, nondivergent wind at 250 mb obtained from inverting (a) analysed potential vorticity (PV) and (b) ozone-derived PV. Contour interval for velocities is 10 m s^{-1}. Vectors are displayed at alternate grid points. A single smoothing pass has been applied to the contours.

Davis et al. (1999), QJRMS, **125**, 3375-3391