ATOC 5050: Atmospheric dynamics

Project 2: Modeling the atmosphere
Due: 5pm, Friday 10 December 2009
(Deliver to David's office or mail box 151 in CIRES)

Group work is encouraged, but the report you hand in should be you own. You should hand in a report of no more than 2-4 pages. Be sure to number your figures, and make reference to these numbers in your text (i.e., Figure 1 shows my best forecast.)

In the mid 1940s it became clear that newly invented computers could be used for the numerically demanding task of predicting weather. Indeed weather prediction was amongst the first uses of computers. We will us a model to do a "hindcast" prediction the storm event of October 2009 using a model very much like that of Charney *et al.*, (1950). The model uses conservation of potential vorticity in non-divergent form, which we suspect is a reasonable framework for considering large-scale atmospheric motions. A model quick tutorial and source code can be obtained from the class web site.

Model background

The vorticity equation can be written for non-divergence barotropic conditions in the mid-latitudes as,

$$\frac{\partial \zeta}{\partial t} = -\left(u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} \right) - v\beta + S(\zeta) \qquad (1)$$

where $\beta = \partial f/\partial y$ is assumed constant for the mid-latitudes, and $S(\zeta)$ can represent any non-conservative sources and sinks of vorticity. Since the flow is assumed non-divergent, we can define a streamfunction, $\psi = gZ/f_0$ and the Laplacian of which is the vorticity. The gradient of which gives u and v. Knowing these relationships and the geopotential height, one can evaluate the right hand side of (1) using finite difference approximations to the derivatives. Similarly the left hand side of (1) can be expressed in finite differences, to obtain a prediction equation for vorticity at some time in the future ($t+\Delta t$) given the vorticity some time in the past ($t-\Delta t$). I.e.,

$$\zeta(t+\Delta t) = \zeta(t-\Delta t) + 2\Delta t\left(\frac{\partial \zeta}{\partial t} \right)_t \qquad (2)$$

Integrating this equation for numerous small time increments ("timesteps") a forecast can be made. This scheme was the basis of the first numerical weather forecast on one of the first computers in the late 1940s. With modern computers and algorithms, this prediction can be done with many fewer problems than the pioneers faced in 1940s.

Further reading

Holton 4.4-4.5, 7.7, Chapter 13 (especially 13.4)
Charney, J. G., Fjortoft, R., and von Neumann, J., Numerical integration of the barotropic vorticity equation. Tellus, 2(4), 1950

Part 1: Rossby waves and a model of phase speed
a) This is a "beta-plane model". What does that mean?
b) Prove (1) is derived from conservation of quasi-geostrophic potential vorticity: $q = \zeta + f$
c) Rewrite (1) in terms of the streamfunction, ψ.

d) Writing each quantity as a mean and a deviation (e.g., $\psi = \overline{\psi} + \psi'$), show that (1) can be written in the perturbation form (i.e., an expression for changes in ψ'). Assume $\bar{v}$ and $\bar{\zeta}$ are zero, $\bar{u}$ is constant and note that products of deviations are small and can be neglected (i.e., $a'b' << ab$):

e) Use this result and assume a wave solutions of the form $\psi' = \mathrm{Re}[Ae^{i\theta}] = |A|\cos(\theta)$, where $\theta = kx+ly-\nu t$, to derive an expression of the phase speed, c, of small (i.e., linear) 2-dimensional waves in a flow with a constant mean zonal flow of $\bar{u}$. (Hint: it should depend on k, l and β)

f) Use the model to test you phase speed expression. Use a set of idealized initial conditions containing a single wave given by k and l, and examine the model output to estimate the phase speed. Make a graph of phase speed versus k for two or three values of l. How do the model values compare with the theoretical expectation? Give a brief explanation of why difference might exist. (Warning: be careful with the units of k and l!)

Part 2: Weather prediction
We will make a forecast of snow storm in Colorado from October 2009. Snow began falling on in the evening of Tuesday 27 October, with campus closed on the 28th, and snow continuing for 3 days to give about 18-24 inches of snow. Use the model to predict the geopotential height field over the US for the ***1200 UTC Wednesday 28 October*** and validate the forecast.

a) How many days prior to the snow storm could the storm be predicted?
Run the model making forecasts for the 28 October starting each of 1day, then 2days, and on successively 1 day earlier to determine how far in advance this model would give warning of the storm. Construct maps of the forecast geopotential height, vorticity and wind, to compare the model simulations against truth, and comment on the differences.

b) The additional source terms in (1) include horizontal dispersion by motions smaller than the numerical grid can resolve, drag on the flow due to contact with the ground, and vorticity generation due to topography:

$$S_\zeta = \kappa\nabla^2\zeta - \gamma\zeta - \frac{f_0}{H}\left(u\frac{\partial Z_s}{\partial x} + v\frac{\partial Z_s}{\partial y}\right) \quad (3)$$

where Z_s is the topographic height. The strength of these can be controlled by the tunable eddy diffusivity κ, drag coefficient γ, and mean fluid depth H. Rerun your 72 hour forecast (i.e., the run starting from 1200 UTC on 25 October 2009), using different values for these. Explain what each does to the results. Can you find a combination of values which gives a better (the best!) forecast? Comment on your findings.
(*Hint: you might start by trying double or half the default values.*)