ATOC 5050: Atmospheric dynamics

Homework assignment 3

Due: 5pm, Tuesday 10 November 2009 (in class, or deliver to David's office)

1.

a. Using the Boussinesq equations on an *f*-plane (i.e., assume constant density, $\rho=\rho_0$, and constant $f=f_0$), show the vertical gradient in the *u* and *v* components of the geostrophic flow vanishes.

$$\frac{\partial u_g}{\partial z} = \frac{\partial v_g}{\partial z} = 0$$

b. This is known as the Taylor-Proudman theorem, which tells us that such flow is purely 2-dimensional. What is the implication for geostrophic flow in the region of topography?

c. Taking $f = 2\Omega\sin\phi$ compute the divergence associated with geotropic flow.

2.

a. Compute the circulation associated with a zonally symmetric wintertime Hadley Cell. Assume the cell extends meridionally from 10°S to 30°N, and vertically from the surface to 200 hPa (about 12km). Mean tropospheric temperature is 275K at 10°S and 260K at 30°N. Surface northerly and upper tropospheric southerly winds have a magnitude of 2 m/s. Vertical velocities are 0.02 Pa s^{-1} (about 0.004 ms^{-1}).

b. Estimate the mean vorticity over this circulating area (careful with units!). Which direction does this vorticity vector point?

c. Contrast the magnitude of (i.e., what is the ratio between) your vorticity with typical values of the vertical component of relative vorticity in mid-latitudes.

d. Using the circulation theorem, estimate the acceleration of the flow near the surface, and determine the wind speed if starting from reset and allowing acceleration for a day.

3.

a. Consider an incompressible fluid in solid body rotation in a cylindrical tank with a flat bottom and a free surface at the upper boundary. Let H be the depth at the center of the tank, Ω the angular velocity of the tank and *a* the radius of the tank. Derive a formula for the dependence of depth on radius.

b. What is the difference in depth from the perimeter to the rotation axis if the fluid has a depth at the center 10 cm and a rotation rate of 20 revolutions per minute and a radius of 50 cm?

c. Compute the relative vorticity of a column of water in the case above if it were initially at rest at the perimeter and moved to the center of the tank.

d. Derive an expression of the tangential velocity, *u*, as a function of radius for this case as columns move toward the interior.

e. Assuming this velocity profile is in gradient wind balance, derive a new expression for the height of the surface as a function of radius.

f. Compare your answer to e that of a. Are they the same? Explain.