

# Turbulence and boundary layers

# Weather and turbulence

*“Big whorls have little whorls which feed on the velocity; and little whorls have lesser whorls and so on to viscosity”*

– Lewis Fry Richardson

# Field trip



- 2 files from tower (wind and temperature)
- 6 separate files (or 1 big file) from balloon
- Couple of IDL examples of how to read the data

# ATOC5050\_20091105\_Wind\_trim.dat

```
"TOA5","CR3000","CR3000","2916","CR3000.Std.08","CPU:MetTower_4sensors.CR3","6847","Wind"  
"TIMESTAMP","RECORD","WS_ms_04","WindDir","WS_ms_03","WS_ms_02","WS_ms_01"  
"TS","RN","meters/second","Degrees","meters/second","meters/second","meters/second"  
"","","Smp","Smp","Smp","Smp","Smp"  
"2009-11-05 09:58:35",0,0,172.4,0,0,0  
"2009-11-05 09:58:40",1,2.6,164.6,2.6,2,2.6  
"2009-11-05 09:58:45",2,1.7,166.1,1.7,1.4,1.85  
"2009-11-05 09:58:50",3,0.95,164.2,1.1,1.25,1.85  
"2009-11-05 09:58:55",4,0.35,162.3,0.5,0.65,1.25  
"2009-11-05 09:59:00",5,1.7,165.7,0.65,0.95,1.1  
"2009-11-05 09:59:05",6,1.85,161.1,1.25,1.55,1.4  
"2009-11-05 09:59:10",7,3.05,174.4,2,1.85,1.85  
"2009-11-05 09:59:15",8,3.2,168.6,2.3,1.85,2.45  
"2009-11-05 09:59:20",9,3.05,168.6,2.3,2,2.45  
"2009-11-05 09:59:25",10,2,166.3,1.55,1.4,2  
"2009-11-05 09:59:30",11,1.55,164.1,0.65,0.95,1.55  
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"2009-11-05 09:59:55",16,1.85,172,1.7,1.55,2.15  
"2009-11-05 10:00:00",17,2.75,170.9,2.3,2,2.3
```

# ATOC5050\_20091105\_Temp\_trim.dat

```
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"TIMESTAMP", "RECORD", "BattV_Min", "T107_C_04", "T107_C_03", "T107_C_02", "T107_C_01"  
"TS", "RN", "Volts", "Deg C", "Deg C", "Deg C", "Deg C"  
"", "", "Min", "Smp", "Smp", "Smp", "Smp"  
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"2009-11-05 09:59:50", 15, 12.68, 22.42, 22.35, 22.6, 22.62  
"2009-11-05 09:59:55", 16, 12.68, 22.4, 22.35, 22.59, 22.61  
"2009-11-05 10:00:00", 17, 12.68, 22.38, 22.33, 22.57, 22.59
```

# Balloon1\_Team1.txt

```
Balloon 1
Team 1 Stephanie et al.
Launch 9:52 AM

Time Azimuth Elevation
0 92 2.00
30 90 59.58
60 76 53.17
90 75 49.33
120 77 47.67
150 80 45.83
180 81 46.33
210 82 46.25
240 83 45.42
270 90 45.00
300 89 45.33
330 90 46.00
360 90 47.58
390 92 48.83
420 94 48.42
450 96 45.92
480 97 43.33
510 98 41.33
```

- Or one excel file with all balloon profiles
- (Should be able to do all of part 2, and 1a-b)

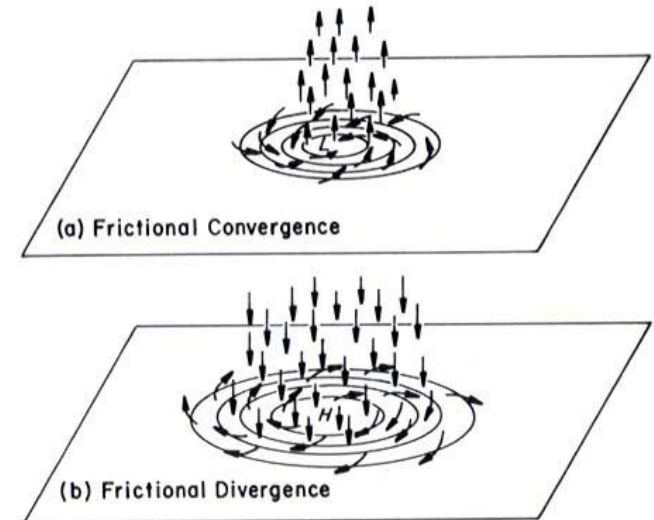
# This class...

- Start to consider role of interaction with the surface
- Role of turbulence in “mixing” momentum, and other stuff (entropy, water, ...)
- Reynolds averaging – how to define the problem
- Richardson number – intuition
- Momentum transfer at the surface (also heat and gas exchange)

# Momentum equations

$$\frac{du}{dt} = fv - \frac{1}{\rho} \frac{\partial p}{\partial x} + Fr_x$$

$$\frac{dv}{dt} = -fu - \frac{1}{\rho} \frac{\partial p}{\partial y} + Fr_y$$



Away from the surface, we can ignore friction

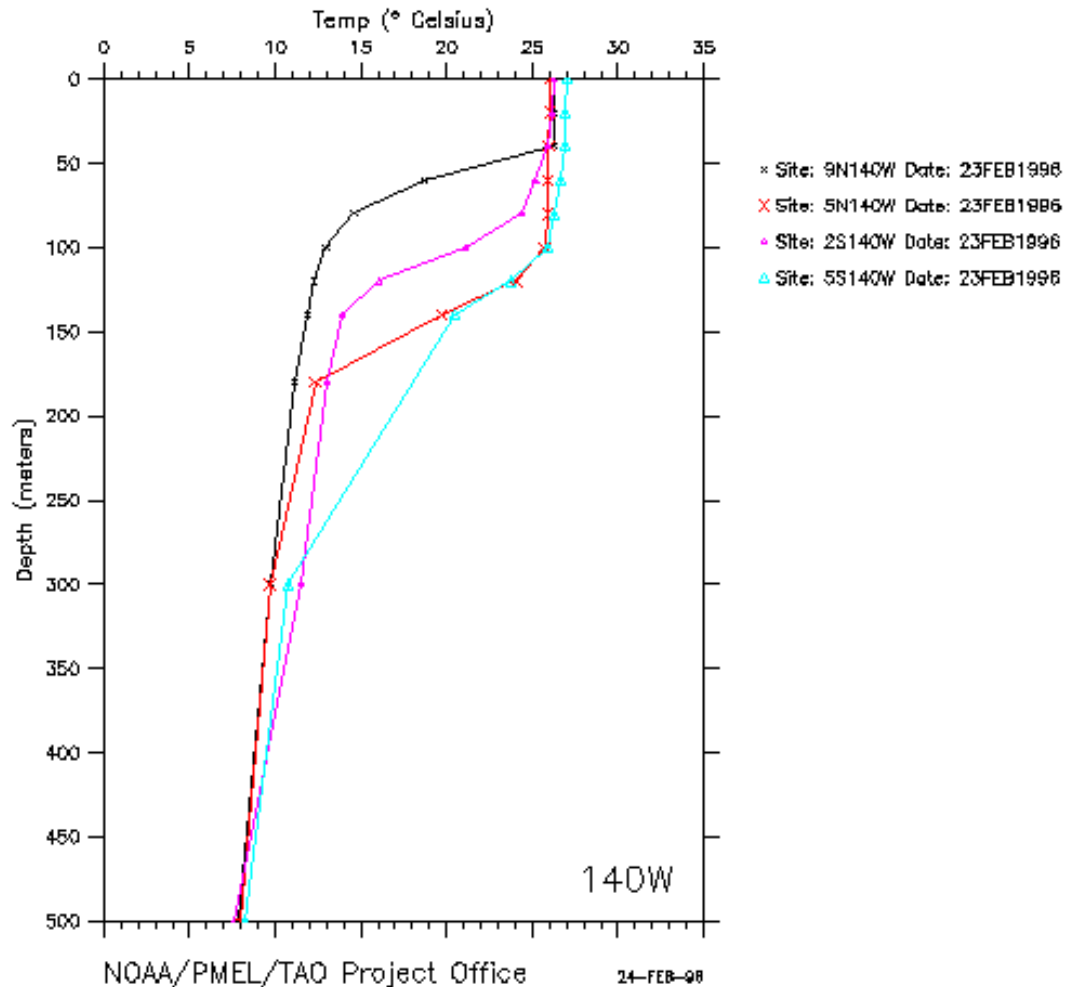
Led to geostrophic assumption

By introducing friction (drag) change the way  
we can balance the equations of motion

# Importance of Turbulent Eddies

- Turbulent eddies are important in the atmospheric boundary layer because they can transport momentum, heat and moisture.
- As a result, the dynamical equations that we have discussed during this semester must be modified for use in the atmospheric boundary layer.
- We will now introduce a strategy for including the effects of turbulence in the dynamical equations.
- To do this, we will attempt to separate the turbulent variations in atmospheric properties from the large-scale variations.

# Vertical structure in the ocean



# Character of turbulence

- The effects of turbulence can be ignored in the free atmosphere, but can not for flow near the surface (e.g., drag ensures flow is ageostrophic)
- Viscosity ensures wind speed is zero very close to the surface
- Turbulent transfer much more efficient than molecular effects (or thermal condition)
- Turbulent eddies exist at all time and space scales between the limits of the boundary layer depth and the scale at which molecular diffusion take over (millimeter)

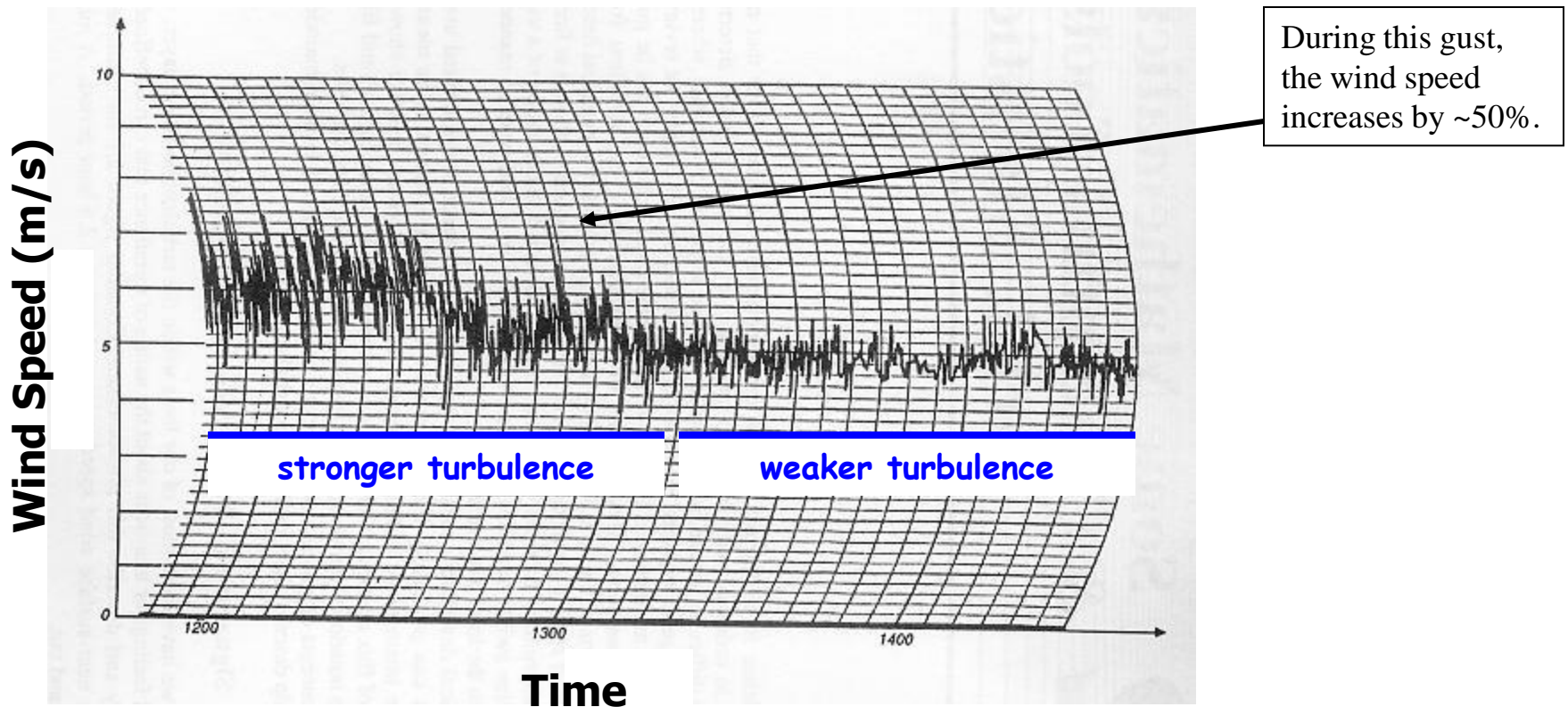


Sonic anemometer

Measures very small  
scale variation in 3d  
wind field

# An Example of Turbulence

The effects of turbulence are evident in this record of surface wind speed measured by an anemometer. The gusts and lulls in the wind, which typically last for less than a minute, are indicative of the passage of turbulent eddies.



# Turbulence

- Eddies “small”, but still much larger than viscous scale
- Energy transferred to smaller scales, where ultimately dissipated by molecular diffusion
- Small scale eddies generated by wind shear  $d|\mathbf{V}|/d\mathbf{x}$  and by buoyancy (i.e., convection)

# Example: Kelvin-Helmholtz

# Equations for turbulence

- We wish to evaluate the “ $Fr$ ” terms in the momentum equation (and equivalent terms in the heat and moisture equations)
- Make use of random nature of turbulent eddies in a statistical representation
- Turbulent eddies small compared to synoptic scale motions (can ignore Coriolis acceleration associated with eddies)
- Also can make some further simplifications of primitive equations for near-surface conditions (Boussinesq assumption, i.e., density pretty constant)

# Boussinesq approximation

- For limited depth region of the atmosphere (say, 1 km), density changes are small compared to the mean density profile.
- Neglect density variations except where they cause buoyant forces
- i.e., given  $\rho = \rho_0 + \Delta\rho$ , use  $\rho_0$  everywhere except for computing buoyancy forces, where we use  $\Delta\rho$

*A particularly appropriate assumption for oceanic flow*

# Boussinesq equations

Horizontal momentum

$$\frac{du}{dt} = fv - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + Fr_x$$

$$\frac{dv}{dt} = -fu - \frac{1}{\rho_0} \frac{\partial p}{\partial y} + Fr_y$$

Vertical momentum  
(non-hydrostatic)

$$\frac{dw}{dt} = -\frac{1}{\rho_0} \frac{\partial p}{\partial z} + g \frac{\Delta\theta}{\theta_0} + Fr_z$$

Thermodynamic

$$\frac{d\Delta\theta}{dt} = -w \frac{d\theta_0}{dz}$$

Continuity

(mean density does not change)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Note  $\theta = \theta_0 + \Delta\theta$

Where  $\theta_0$  is appropriate given  $\rho_0$

# Reynolds averaging

- Define all quantities to be composed of a time mean and a deviation

$$u = \bar{u} + u'$$

$$v = \bar{v} + v'$$

$$w = \bar{w} + w'$$

$$\theta = \bar{\theta} + \theta'$$

At a given point, this deviation from the time mean gives a measure of turbulent eddies

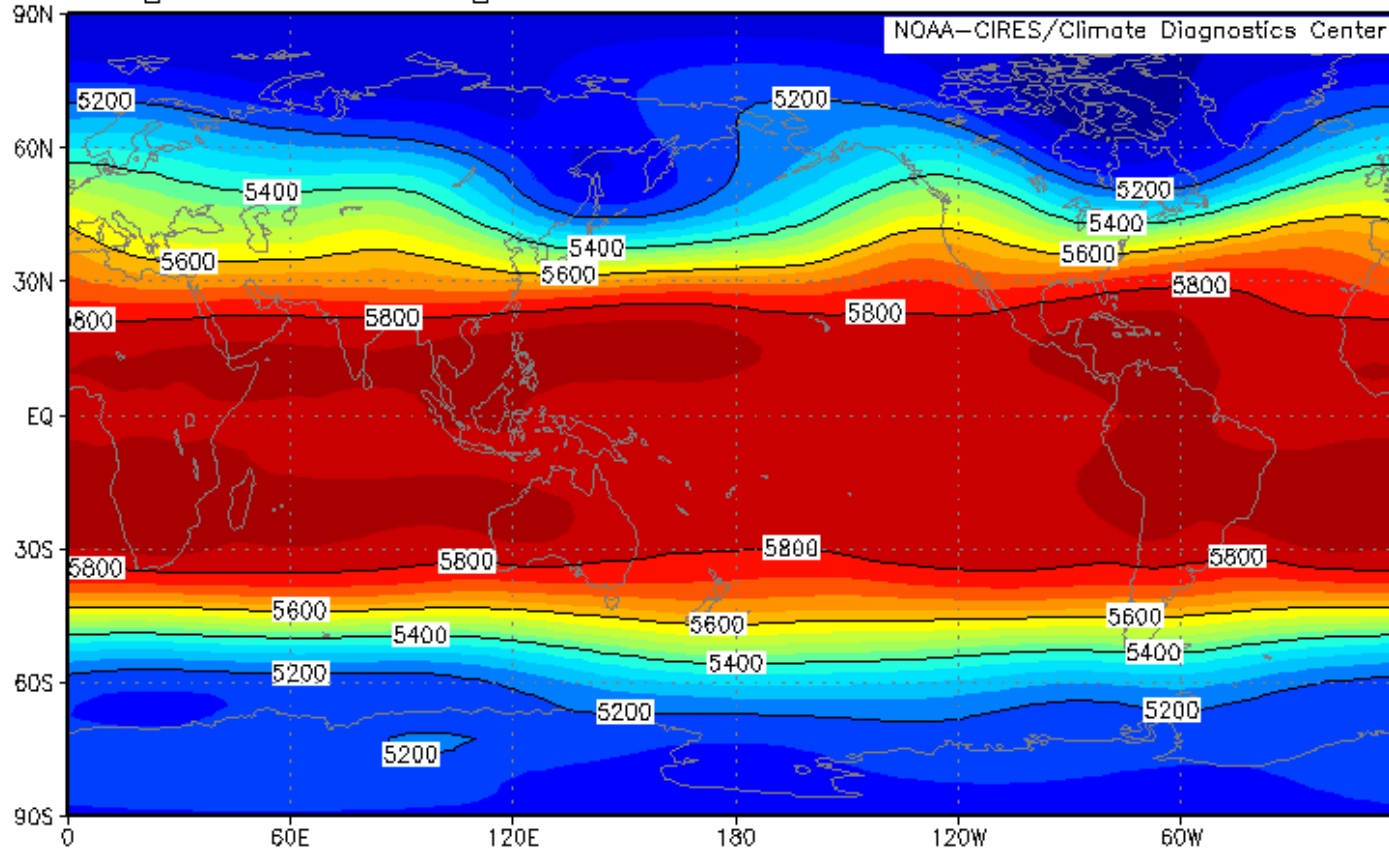
Compare this with spatial deviations we examined with waves

# Stationary (topographically forced) waves

lon: plotted from 0.00 to 357.50  
lat: plotted from -90 to 90.00  
lev: 500.00  
t: Jan

NCEP Reanalysis Z500  
January mean

Long Term Mean hgt m



MAX=5882.66  
MIN=5009.5

GrADS image



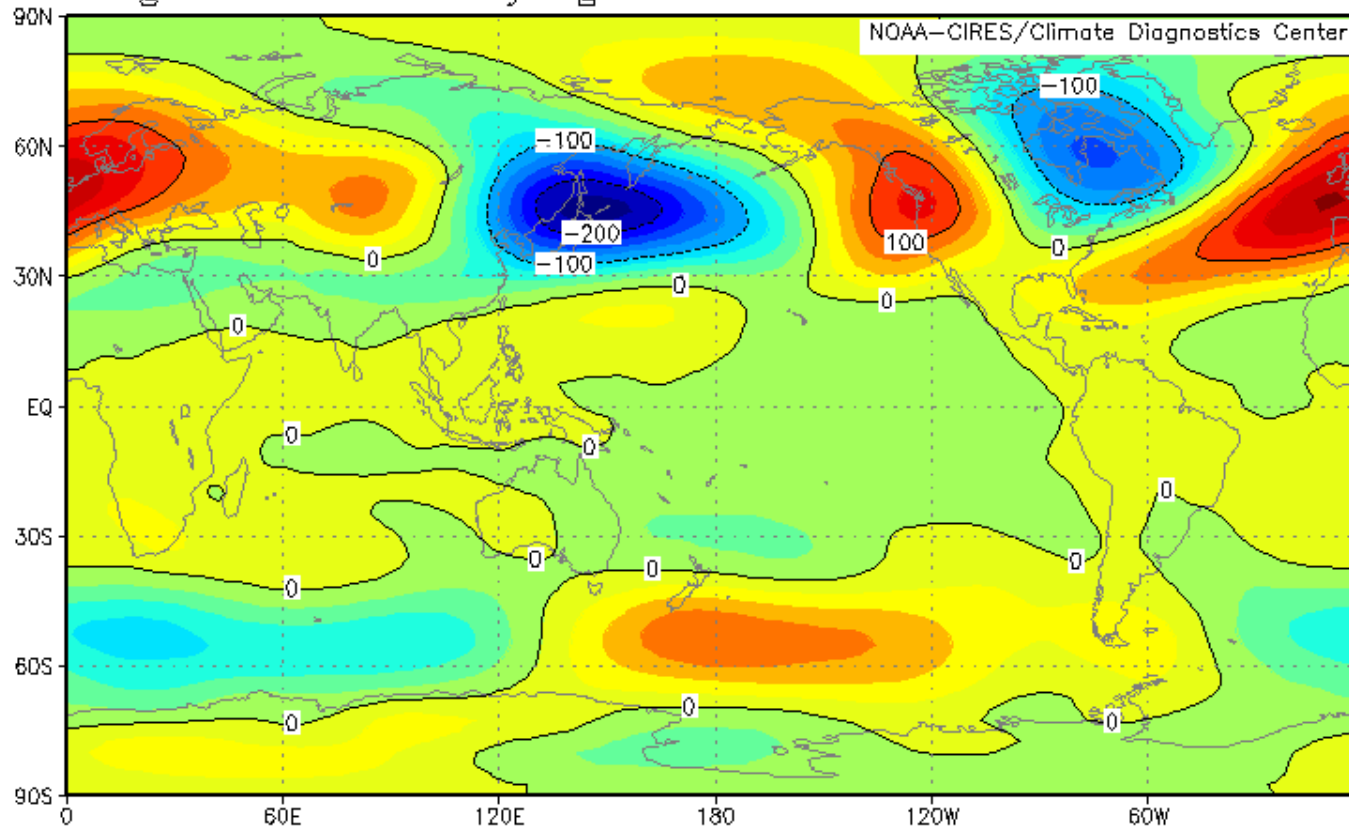
# NCEP Reanalysis Z500

## Deviations from zonal mean

### January mean

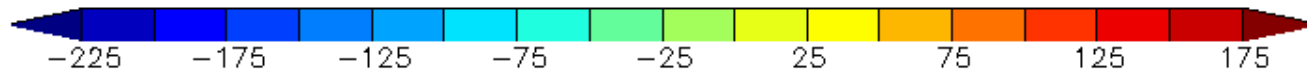
lon: plotted from 0.00 to 357.50  
lat: plotted from -90 to 90.00  
lev: 500.00  
t: Jan

Long Term Mean Eddy hgt m

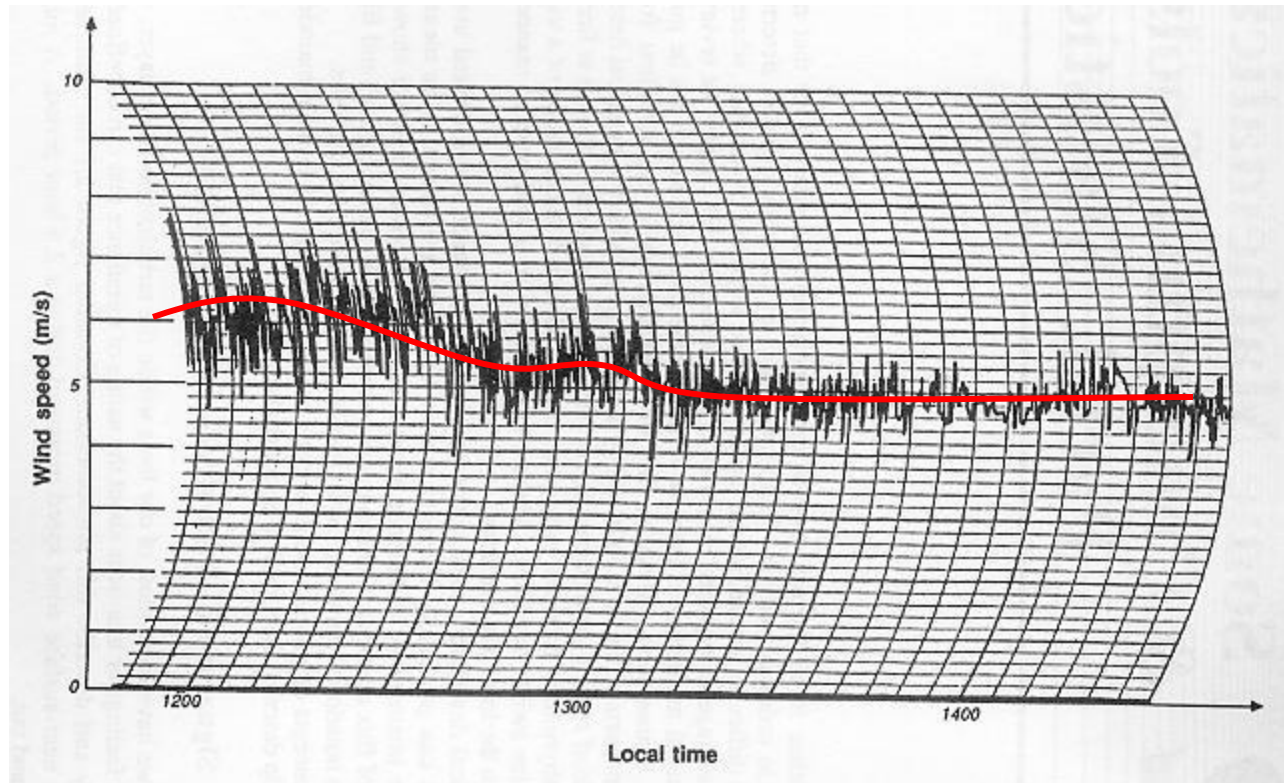


MAX=177.782  
MIN=-237.12

GrADS image



# Separate eddy variations from background

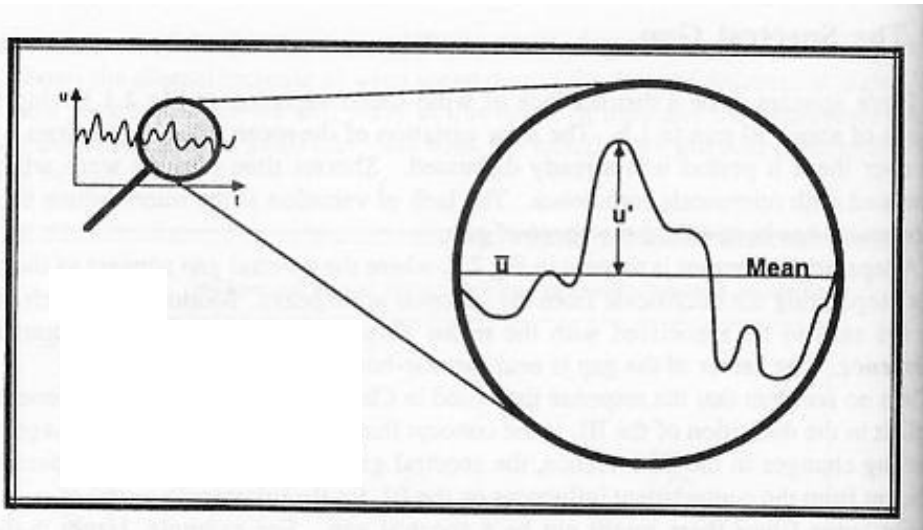


A simple way to separate the turbulent variations from the large-scale variations is to average our wind measurements over a period of 30-60 minutes.

# Turbulent variations

Instantaneous velocities can be decomposed into mean and turbulent components:

$$u = \bar{u} + u'$$



$$u' = u - \bar{u}$$

turbulent velocity      instantaneous velocity      mean velocity

Turbulent velocities are the positive and negative deviations of the instantaneous velocities about the mean.

# Rules of averaging

To include the effects of turbulence in the primitive equations, we need to have some basic rules for dealing with the mathematics of averaging.

Computing the time average of a variable  $A(x,y,z,t)$  that is a function of space and time:

$$\bar{A}(x, y, z) = \frac{1}{N} \sum_{i=0}^{N-1} A(x, y, z, i) \quad \text{discrete function}$$

$$\bar{A}(x, y, z) = \frac{1}{P} \int_{t=0}^P A(x, y, z, t) dt \quad \text{continuous function}$$

# Averaging

Let  $A$  and  $B$  be two variables that vary over time and let  $c$  represent a constant. We will show that:

$$\overline{(A + B)} = \bar{A} + \bar{B}$$

Computing the time average of a variable  $A(x, y, z, t)$  that is a function of space and time:

$$\begin{aligned}\overline{(A + B)} &= \frac{1}{N} \sum_{i=0}^{N-1} (A_i + B_i) \\ &= \frac{1}{N} \left( \sum_i A_i + \sum_i B_i \right) \\ &= \frac{1}{N} \sum_i A_i + \frac{1}{N} \sum_i B_i \\ &= \bar{A} + \bar{B}\end{aligned}$$

discrete

$$\begin{aligned}\overline{(A + B)} &= \frac{1}{P} \int_{t=0}^P (A + B) dt \\ &= \frac{1}{P} \left( \int_{t=0}^P A dt + \int_{t=0}^P B dt \right) \\ &= \frac{1}{P} \int_{t=0}^P A dt + \frac{1}{P} \int_{t=0}^P B dt \\ &= \bar{A} + \bar{B}\end{aligned}$$

continuous

# More averaging

Through similar mathematical manipulation, we can derive the following averaging rules:

$$\overline{c} = c$$

$$\overline{(cA)} = c\overline{A}$$

$$\overline{(\overline{A})} = \overline{A}$$

$$\overline{(\overline{AB})} = \overline{A}\overline{B}$$

$$\overline{(A + B)} = \overline{A} + \overline{B}$$

$$\overline{\left(\frac{dA}{dt}\right)} = \frac{d\overline{A}}{dt}$$

Next we will apply these rules to variables that are split into mean and turbulent components.

# Reynolds Averaging

$$\text{Let } A = \bar{A} + a' \text{ and } B = \bar{B} + b'$$

What is the average of  $A$ ?

$$\bar{A} = \overline{(\bar{A} + a')} = \overline{(\bar{A})} + \bar{a}' = \bar{A} + \bar{a}'$$

In order for the above to be true, then  $\bar{a}' = 0$

Notice important outcome that the mean of the deviations is zero

(sum of positive deviations must equal the sum of the negative deviations)

# Averaging products

$$\begin{aligned}\overline{(AB)} &= \overline{(\bar{A} + a')(\bar{B} + b')} \\ &= \overline{\bar{A}\bar{B} + a'\bar{B} + \bar{A}b' + a'b'} \\ &= \overline{\bar{A}\bar{B}} + \overline{a'\bar{B}} + \overline{\bar{A}b'} + \overline{a'b'}\end{aligned}$$

What is the average of the product of  $A$  and  $B$ ?

However,  $\overline{a'\bar{B}} = \bar{a}'\bar{B} = 0 \cdot \bar{B} = 0$  and  $\overline{\bar{A}b'} = \bar{A}\bar{b}' = \bar{A} \cdot 0 = 0$

$$\begin{aligned}\overline{(AB)} &= \bar{A}\bar{B} + 0 + 0 + \overline{a'b'} \\ &= \bar{A}\bar{B} + \overline{a'b'}\end{aligned}$$

Note that the second term on the right (nonlinear product or cross product) is not necessarily zero.

$$\overline{a'b'} \neq \bar{a}'\bar{b}' \quad \text{Note that} \quad \overline{a'b'} = \frac{1}{N} \sum_{i=0}^{N-1} a'_i b'_i$$

# Reynolds averaging governing equations

- Substitute into momentum equations:

$$\frac{d(\bar{u} + u')}{dt} = f(\bar{v} + v') - \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} - \frac{1}{\rho_0} \frac{\partial p'}{\partial x}$$
$$\frac{d\bar{u}}{dt} + \frac{du'}{dt} = f\bar{v} + fv' - \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x}$$

Noticing advection can be written in the form

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$
$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial(uu)}{\partial x} + \frac{\partial(vu)}{\partial y} + \frac{\partial(wu)}{\partial z}$$
$$\frac{d\bar{u}}{dt} = \frac{\partial \bar{u}}{\partial t} + \frac{\partial(\bar{u}\bar{u})}{\partial x} + \frac{\partial(\bar{v}\bar{u})}{\partial y} + \frac{\partial(\bar{w}\bar{u})}{\partial z} + \frac{\partial(\overline{u'u'})}{\partial x} + \frac{\partial(\overline{v'u'})}{\partial y} + \frac{\partial(\overline{w'u'})}{\partial z}$$

Use fact that mean of mean times deviation is zero

# Turbulent terms

Expanding, and taking the time mean of all terms,

$$\frac{\partial \bar{u}}{\partial t} = +f\bar{v} - \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} - \left[ \frac{\partial(\bar{u}\bar{u})}{\partial x} + \frac{\partial(\bar{v}\bar{u})}{\partial y} + \frac{\partial(\bar{w}\bar{u})}{\partial z} \right] - \left[ \frac{\partial(u'u')}{\partial x} + \frac{\partial(v'u')}{\partial y} + \frac{\partial(w'u')}{\partial z} \right]$$

So, defining  $Fr$

$$\frac{d\bar{u}}{dt} = +f\bar{v} - \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + Fr_x$$
$$Fr_x = - \left[ \frac{\partial(u'u')}{\partial x} + \frac{\partial(v'u')}{\partial y} + \frac{\partial(w'u')}{\partial z} \right]$$

We have an expression of the frictional term

In general the  $z$  component is largest, and of most interest

# Turbulent governing equations

$$\frac{d\bar{u}}{dt} = f\bar{v} - \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} - \left[ \frac{\partial(\overline{u'u'})}{\partial x} + \frac{\partial(\overline{v'u'})}{\partial y} + \frac{\partial(\overline{w'u'})}{\partial z} \right]$$

$$\frac{d\bar{v}}{dt} = -f\bar{u} - \frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial y} - \left[ \frac{\partial(\overline{u'v'})}{\partial x} + \frac{\partial(\overline{v'v'})}{\partial y} + \frac{\partial(\overline{w'v'})}{\partial z} \right]$$

$$\frac{d\bar{w}}{dt} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial z} + g \frac{\overline{\Delta\theta}}{\theta_0} - \left[ \frac{\partial(\overline{u'w'})}{\partial x} + \frac{\partial(\overline{v'w'})}{\partial y} + \frac{\partial(\overline{w'w'})}{\partial z} \right]$$

$$\frac{\partial \overline{\Delta\theta}}{\partial t} = -\bar{w} \frac{d\theta_0}{dz} - \left[ \frac{\partial(\overline{u'\theta'})}{\partial x} + \frac{\partial(\overline{v'\theta'})}{\partial y} + \frac{\partial(\overline{w'\theta'})}{\partial z} \right]$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$

Note  $\theta' = (\Delta\theta)'$

# Example: kinetic energy

$$E = \frac{1}{2} [u^2 + v^2 + w^2] \quad \text{per unit mass}$$

Substituting for mean and deviations,

$$E = \bar{E} + e = \frac{1}{2} [(\bar{u} + u')^2 + (\bar{v} + v')^2 + (\bar{w} + w')^2]$$

Collecting terms, and taking time average,

(notice mean of mean times deviation is zero)

$$\overline{\bar{E} + e} = \frac{1}{2} [\overline{\bar{u}^2 + \bar{v}^2 + \bar{w}^2}] + \frac{1}{2} [\overline{u'^2 + v'^2 + w'^2}]$$

Mean kinetic energy

$$\bar{E} = \frac{1}{2} [\bar{u}^2 + \bar{v}^2 + \bar{w}^2]$$

Turbulent kinetic energy

$$\bar{e} = \frac{1}{2} [\overline{u'^2 + v'^2 + w'^2}]$$

# Diurnal TKE development

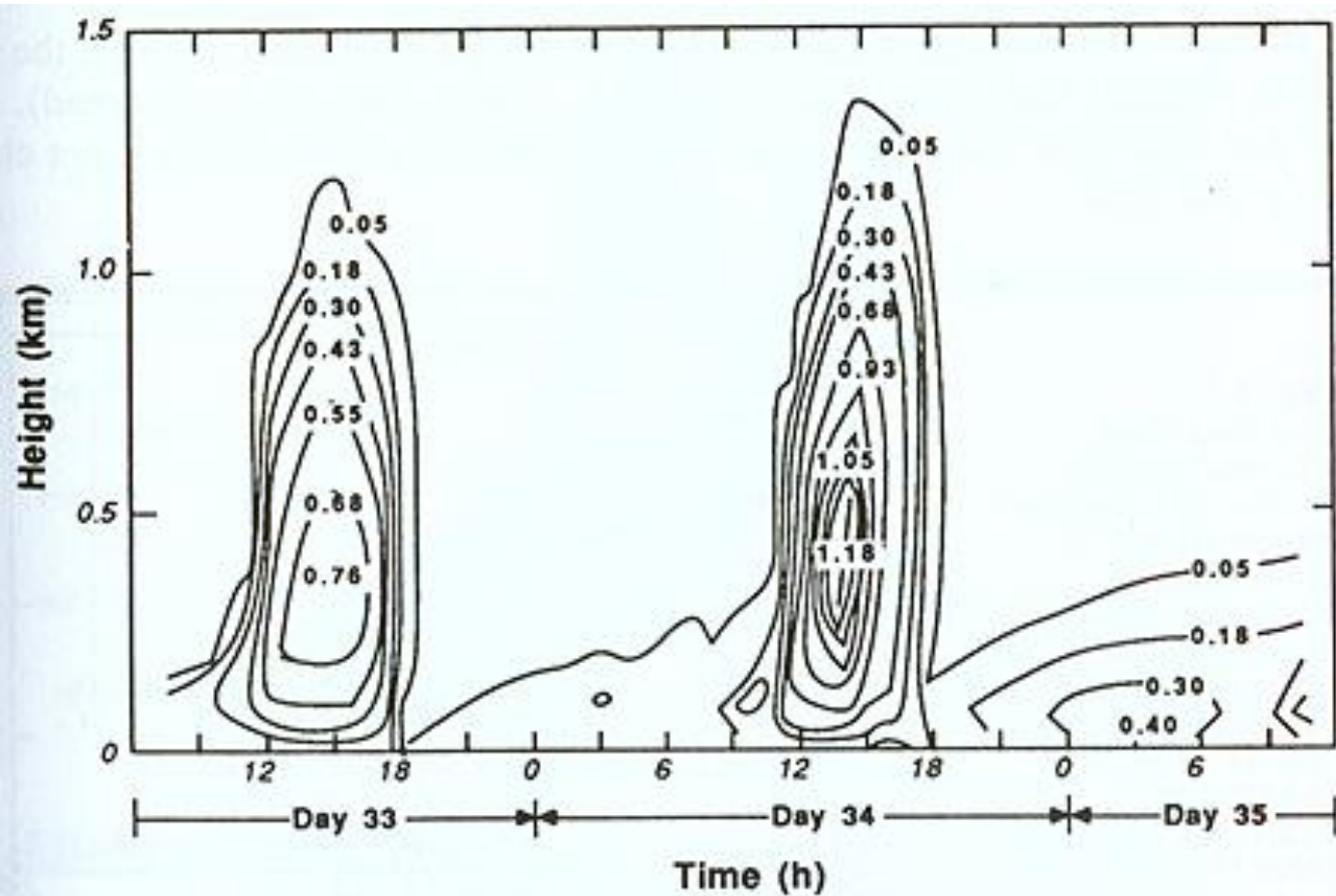


Fig. 5.1 Modeled time and space variation of  $\bar{\epsilon}$  (turbulence kinetic energy, units  $\text{m}^2\text{s}^{-2}$ ), for Wangara. From Yamada and Mellor (1975).

# TKE equation

- Evolution of TKE  $\bar{e} = \frac{1}{2} [\overline{u'^2} + \overline{v'^2} + \overline{w'^2}]$
- Use eddy form of governing equations to derive TKE equation

$$\frac{d\bar{e}}{dt} = \underbrace{-\overline{u'w'}}_A \frac{\partial \bar{u}}{\partial z} - \underbrace{\overline{v'w'}}_B \frac{\partial \bar{v}}{\partial z} + \underbrace{\overline{w'\theta'}}_C \frac{g}{\theta_0} + \underbrace{\frac{\partial(\overline{w'e})}{\partial z}}_D + \underbrace{\frac{1}{\rho_0} \frac{\partial(\overline{w'p'})}{\partial z}}_E - \underbrace{\varepsilon}_F$$

A: local change and advection by the mean flow

B: mechanical production due to wind shear

C: production by buoyancy (convection)

D: transport by eddies

E: redistribution by pressure (gravity) waves

F: dissipation, conversion of mechanical energy to heat

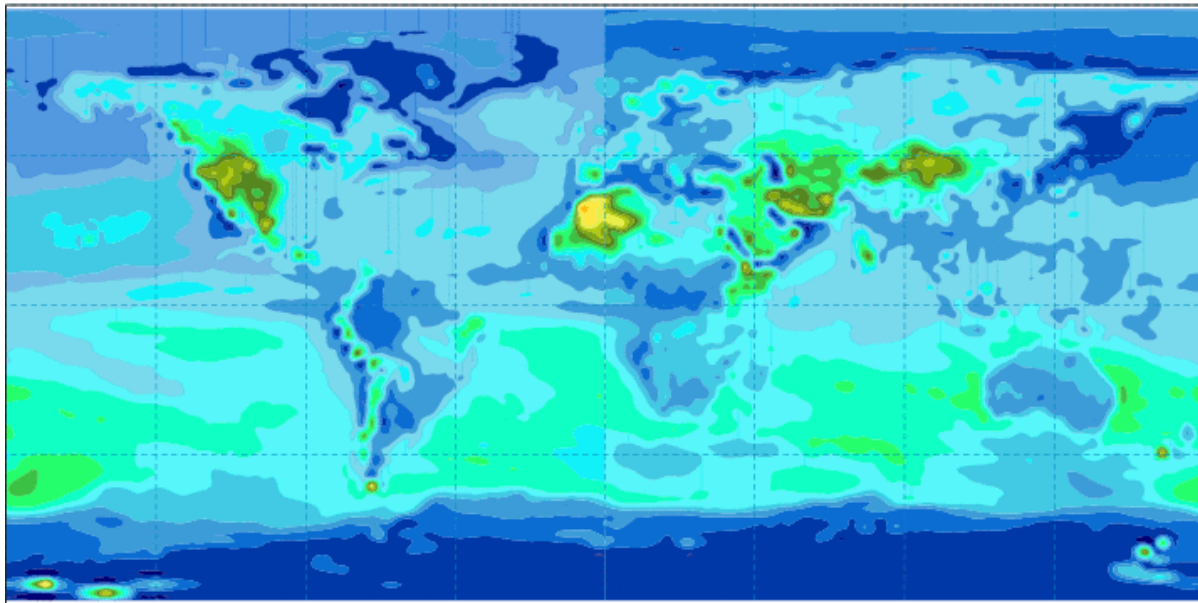
# Flux Richardson number

Ratio of buoyant and shear eddy generation

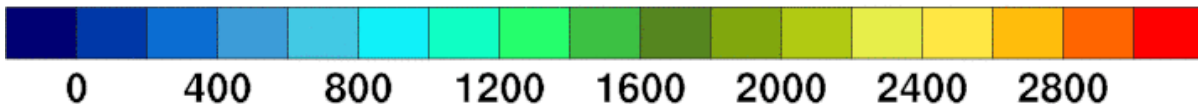
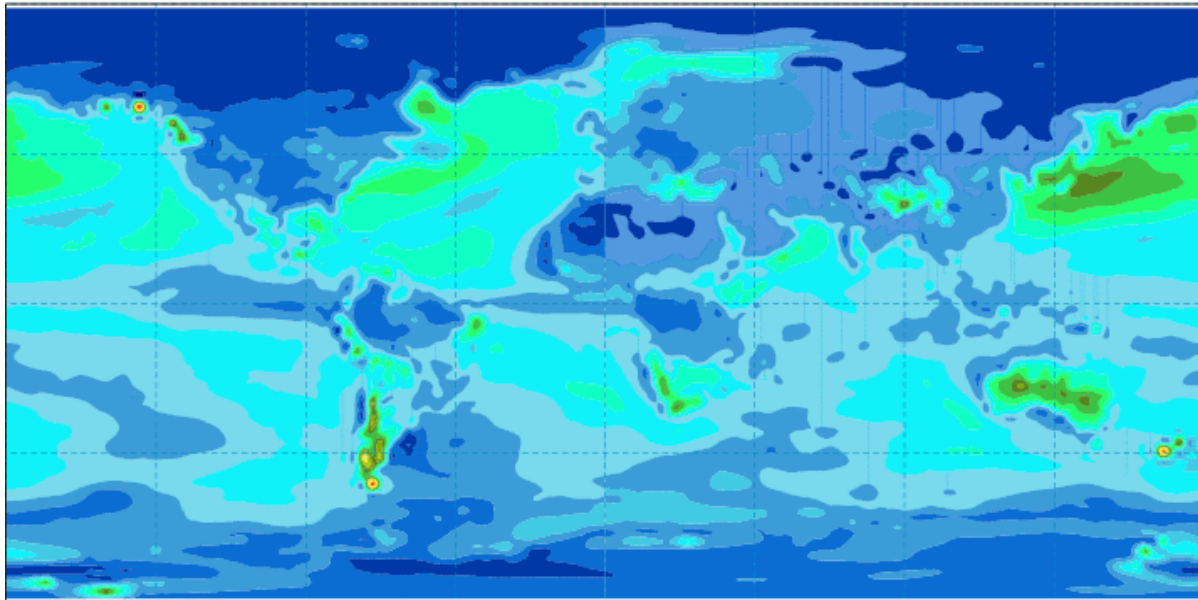
$$R_f = \frac{\text{buoyancy production}}{\text{mechanical production}} = \left( \frac{\overline{w'\theta'} g}{\theta_0} \right) \left[ -\overline{u'w'} \frac{\partial \bar{u}}{\partial z} - \overline{v'w'} \frac{\partial \bar{v}}{\partial z} \right]^{-1}$$

- A measure of the stability, or degree to which the atmosphere wants to resist developing turbulence
- $R_f > 1$  stability dominates and flow is laminar (turbulence tends to decay)
- $R_f < 1$  flow unstable and turbulence develops (eddies generated by strong wind shear)
- Can be used to define top of the boundary layer, by looking for altitude where  $R_f = 1$

ERA40 PBLH [m] July 2000



ERA40 PBLH [m] January 2000



# Equations for the boundary layer

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} - f\bar{v} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} - \frac{\partial}{\partial z} (\overline{u'w'})$$
$$\frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} + f\bar{u} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - \frac{\partial}{\partial z} (\overline{v'w'})$$

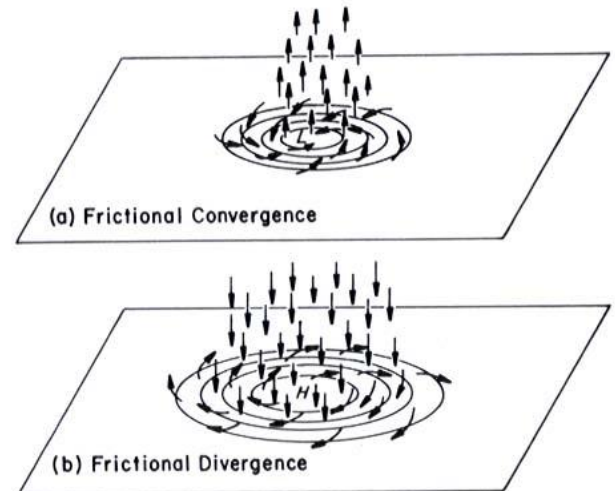
Simplify for balanced flow

(making use of geostrophic definition),

$$f(\bar{v} - \bar{v}_g) = \frac{\partial}{\partial z} (\overline{u'w'})$$

$$f(\bar{u} - \bar{u}_g) = -\frac{\partial}{\partial z} (\overline{v'w'})$$

States that turbulent transfer is responsible for a geostrophic flow



# Challenge

- Have our boundary layer equations, which are (more or less) exact.
- But we don't know how to evaluate the eddy terms
- Challenge is to derive a theory, which can be used to express eddy terms as a function of mean terms

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} - f\bar{v} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} - \frac{\partial}{\partial z} (\overline{u'w'})$$
$$\frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} + f\bar{u} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial y} - \frac{\partial}{\partial z} (\overline{v'w'})$$

## 2 outcomes:

- 1) Be able to estimate momentum, heat, water flux  
(useful from field measurements, also in models)
- 2) Provide simplified depiction of role of friction on large scale dynamics