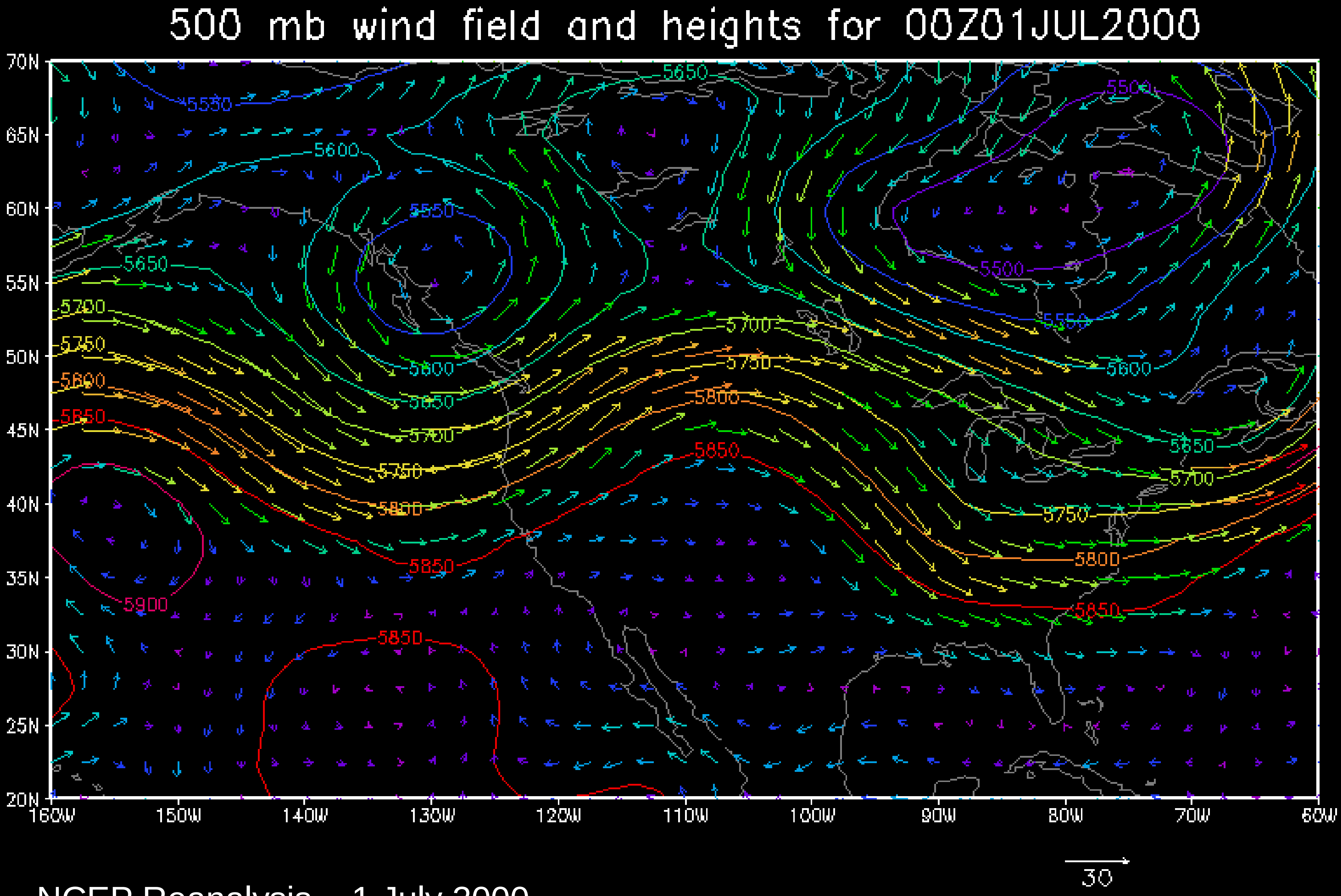
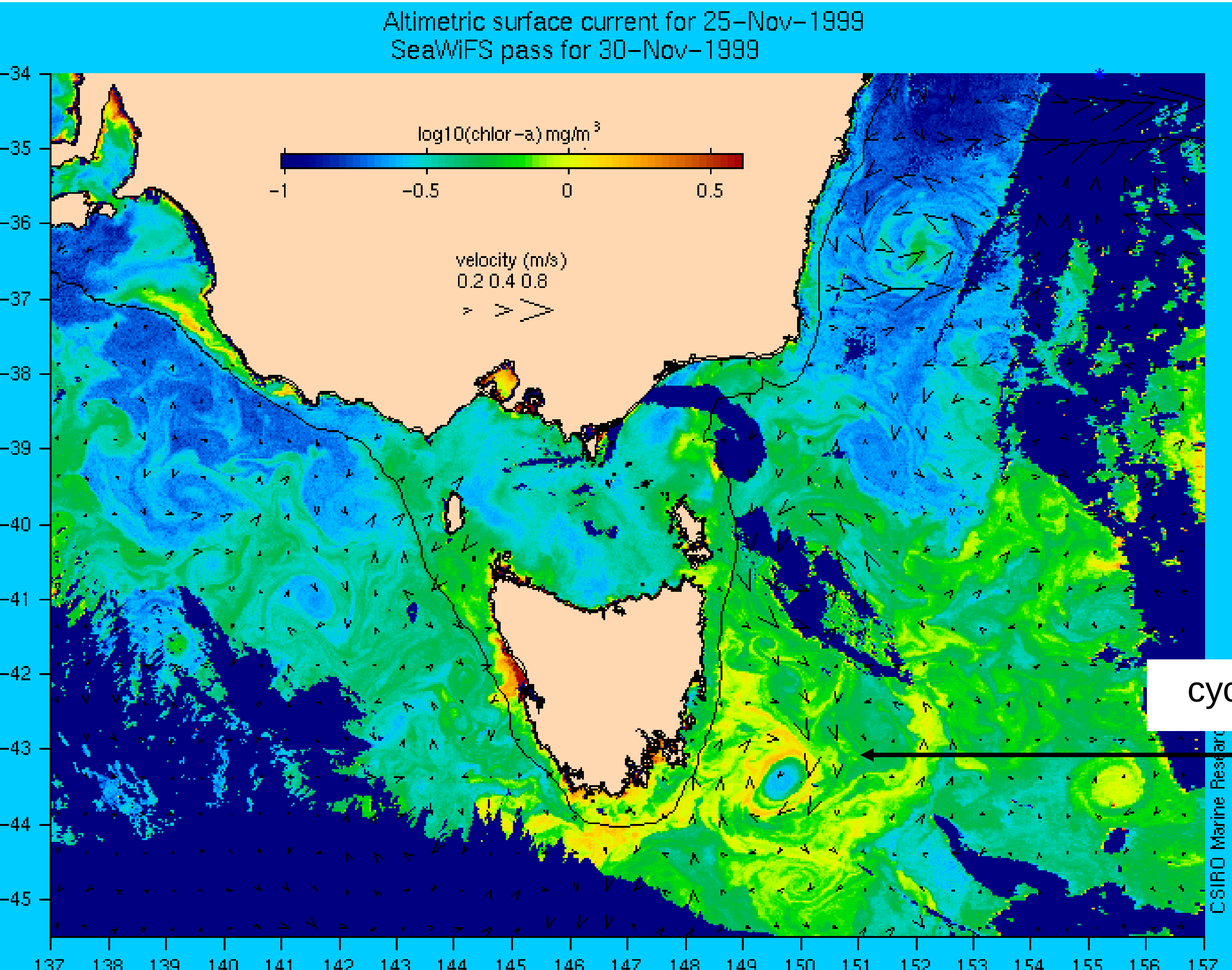
Thermal wind balance

Large scale flow almost geostrophic



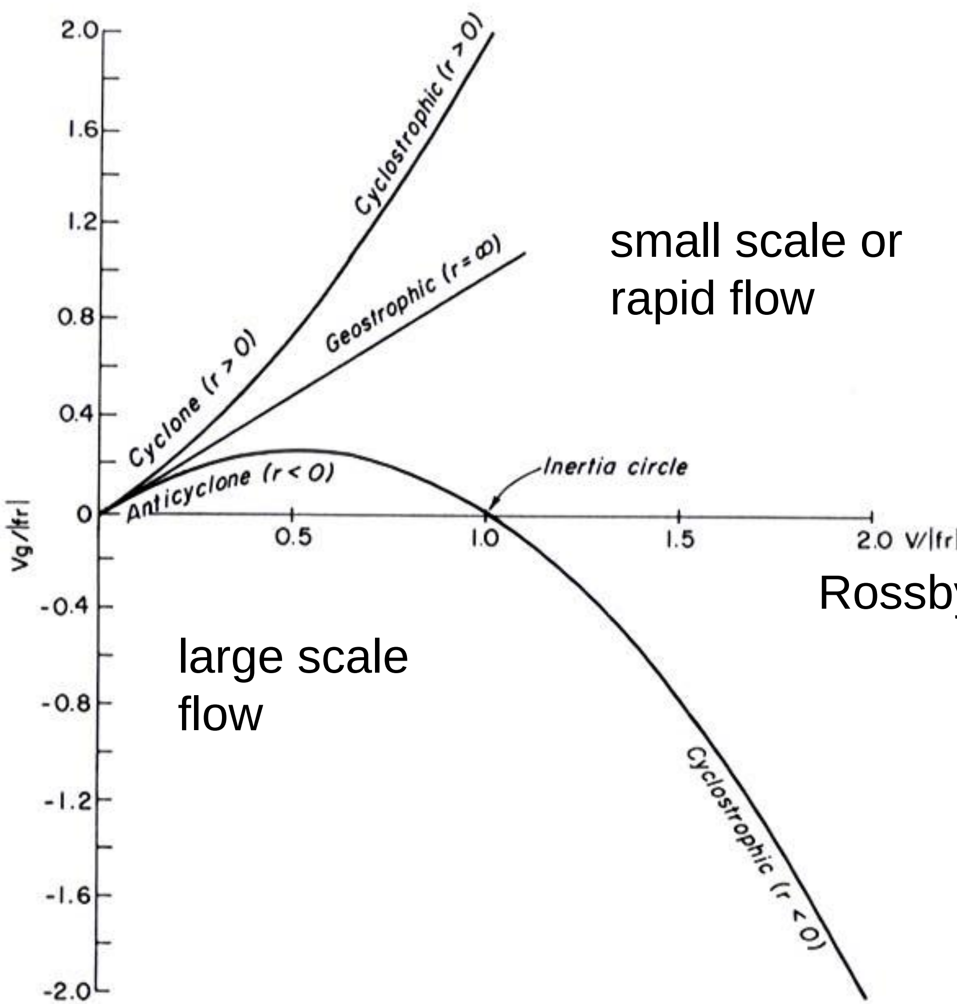
NCEP Reanalysis – 1 July 2000



cyclostrophic

Hurricanes also almost cyclostrophic

Comparison of balanced flows



small scale or
rapid flow

Rossby number

large scale
flow

FIG. 4.8. Gradient wind diagram for circular flow in the counterclockwise direction ($r > 0$) and the clockwise direction ($r < 0$) in the Northern Hemisphere.

Fleagle and Businger, An introduction to Atmospheric Physics, pg. 164

Hydrostatic and geostrophic

- A match made in …err… a GFD lab?

Thermal wind

- From hydrostatic (hypsometric) equation, we know changes in high related to mean temperature
- Consider the difference in geostrophic flow at two levels:

$$(V_g)_2 - (V_g)_1 = \frac{1}{f}\frac{\partial}{\partial n}(\Phi_2 - \Phi_1)$$

$$(\Phi_2 - \Phi_1) = R\overline{T}\ln\frac{p_1}{p_2}$$

$$V_T = (V_g)_2 - (V_g)_1 = \frac{1}{f}\frac{\partial}{\partial n}(\Phi_2 - \Phi_1)$$

$$V_T = \frac{R}{f}\frac{\partial \overline{T}}{\partial n}\ln\frac{p_1}{p_2}$$

•Thermal wind (V_T) defined as a (vector) difference in geostrophic wind with height (i.e., geostrophic with shear)
•Robust result as from hydrostatic and geostrophic assumptions

Example

Q: NH mid-latitude temperature gradient is about 1K/1000km. The zonal mean zonal wind speed is about 5 m/s. What is the zonal wind speed at the tropopause?

A: $$u_T = \Delta u_g = \frac{R}{f}\frac{\partial \overline{T}}{\partial y}\ln\frac{p_1}{p_2}$$

$$u_g(1000) - u_g(250) = \frac{R}{f}\frac{\partial \overline{T}}{\partial y}\ln\left(\frac{p_{1000}}{p_{250}}\right)$$

So, u_g(250) = 5 + 35.8 ~ 41 m/s

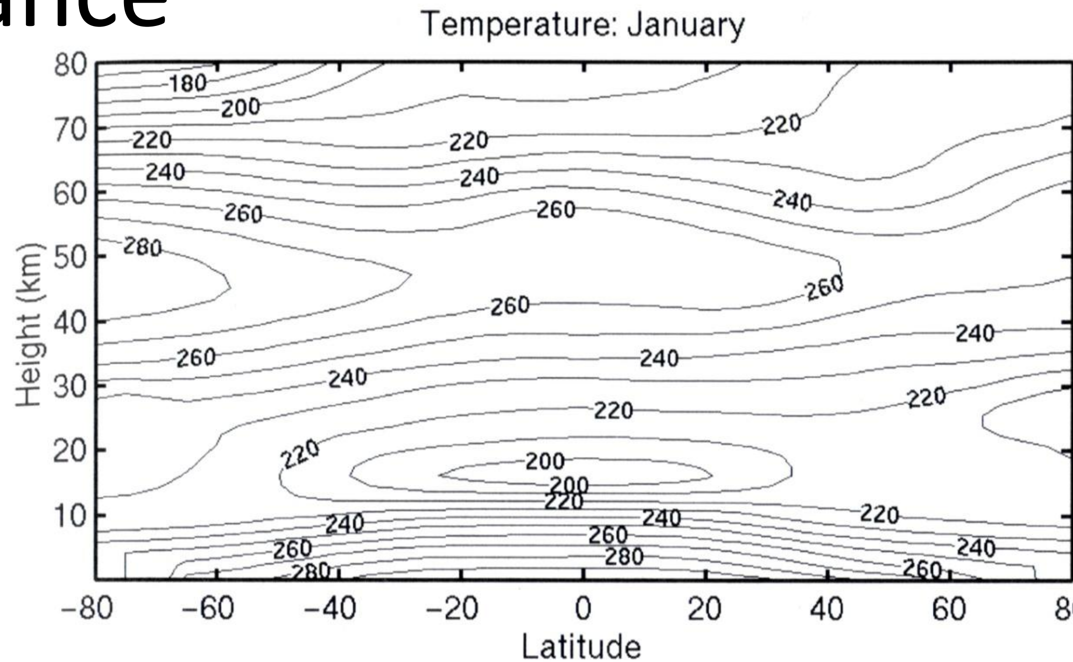
Thermal wind balance

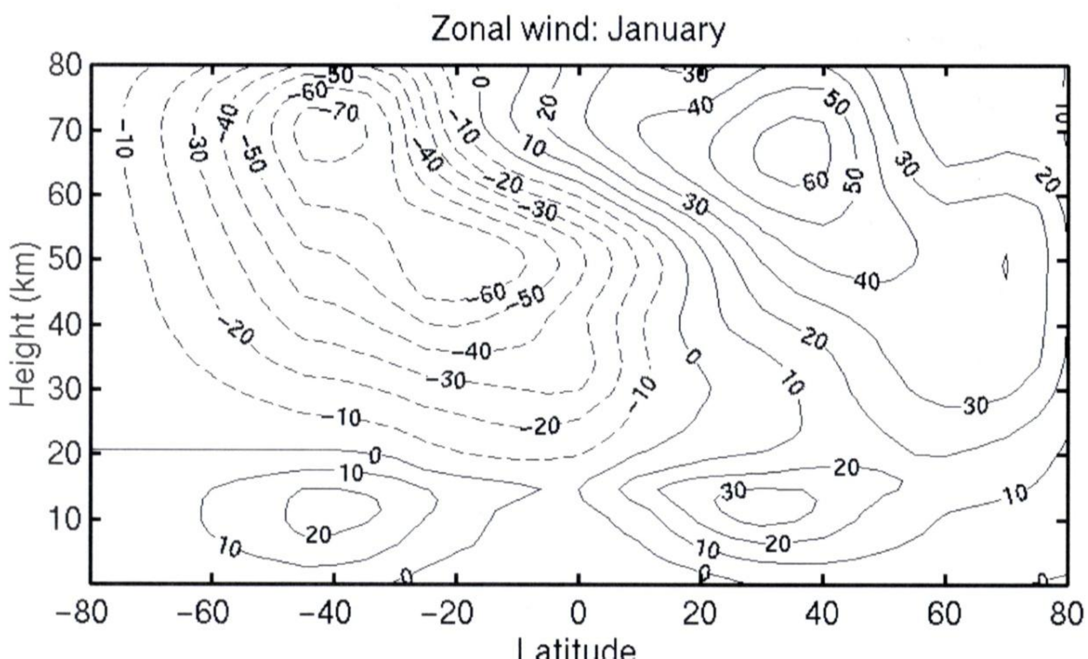
For zonal mean, we can write

$$\frac{\partial u_g}{\partial z} = -\frac{R}{fH}\frac{\partial T}{\partial y}$$

As before, vertical (geostrophic) wind shear related to horizontal temperature gradient.

Since geostrophic and hydrostatic assumptions are robust, this is also a robust association,
one of the most important equations for large scale meteorology





Thermal wind vector

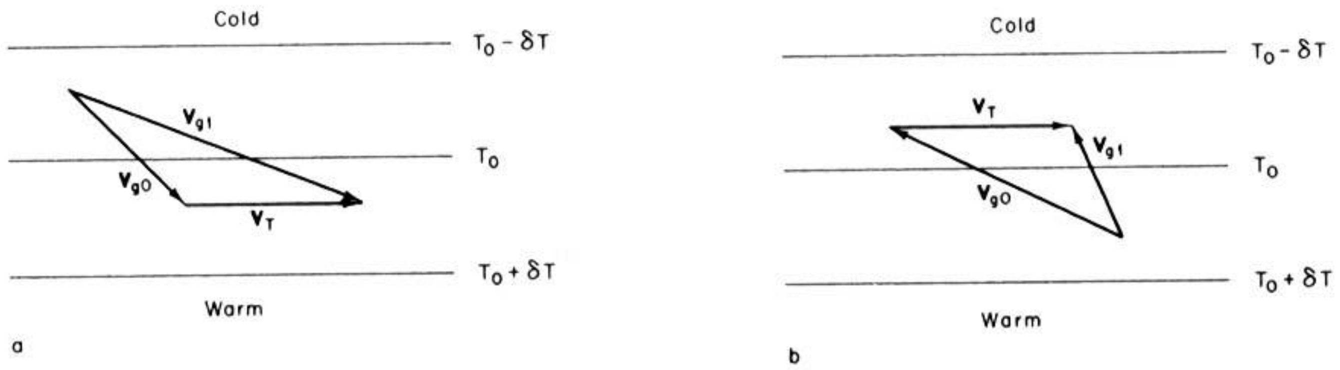
In vector form,

$$\mathbf{V}_T = -\frac{R}{f}\ln\left(\frac{p_1}{p_2}\right)\left(\mathbf{k}\times\nabla\overline{T}\right)$$

Thus parallel to isotherms (analogous to geostrophic wind for pressure contours).
Recall form of geostrophic wind,

$$\mathbf{V}_g = \frac{1}{f}\left(\mathbf{k}\times\nabla\Phi\right)$$

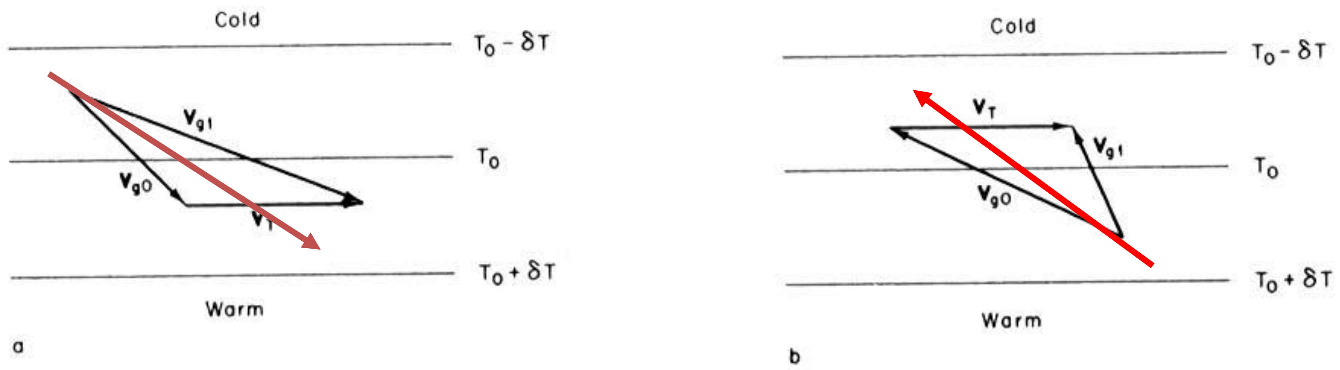
For thermal wind, cold on the left in NH and on the right in SH



Possible to determine (3d) geostrophic wind field knowing the 3d temperature field and surface geostrophic flow
(very useful if you can measure temperature by balloon or satellite sounding)

Implications diagnosis

- ***Geostrophic temperature advection leads to changes in wind speed with height***

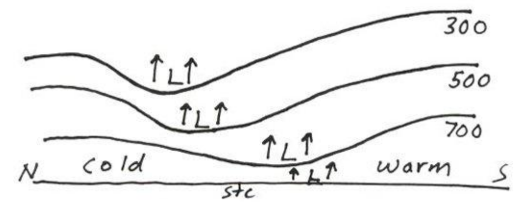


Cold advection causes backing (same as planetary rotation)
Warm advection causes veering with height

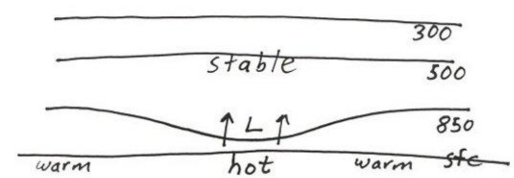
You observe low clouds moving west, and high clouds moving south. What prediction do you make?

Implications for cyclones

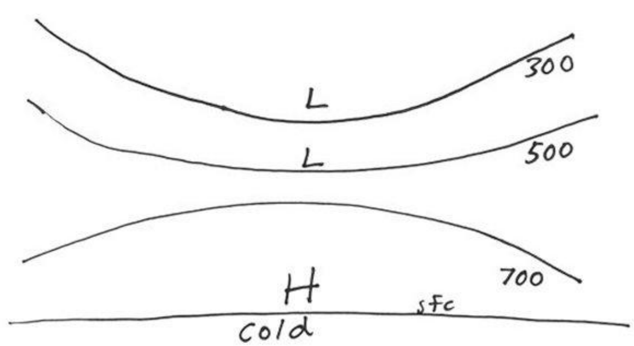
- Cold core lows intensify upward (e.g., extratropical cyclones)

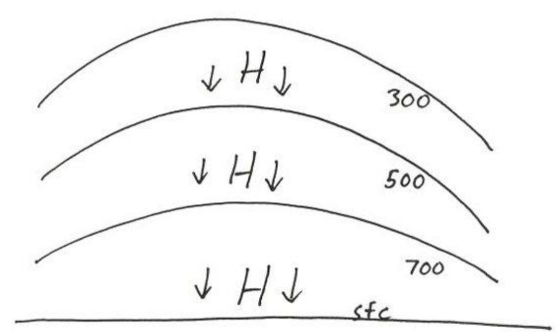


- Warm core lows weaken upward, and can in fact reverse (e.g. hurricanes)



- Also for highs…





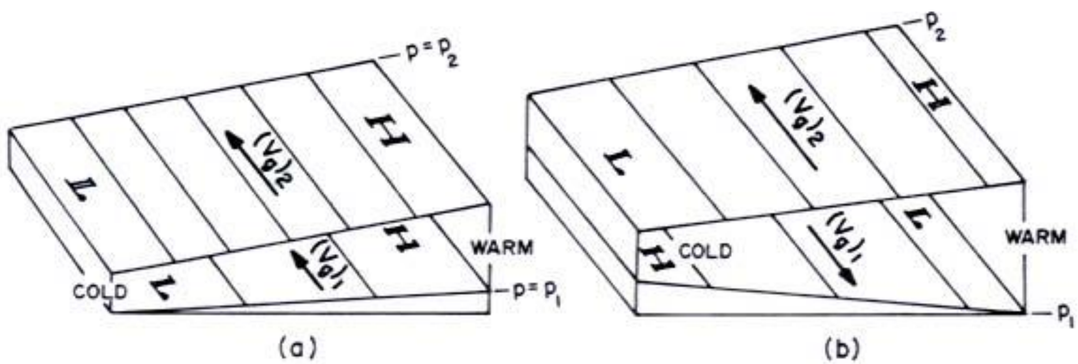
Barotropic and baroclinic atmospheres

Consider orientation of temperature contours relative to height contours.

- Barotropic
  - no horizontal (pressure) temperature gradient
  - geostrophic flow independent of height
- Equivalent barotropic
  - height and thickness contours parallel
  - geostrophic flow changes speed with height
- Baroclinic
  - temperature variations exist without restriction
  - geostrophic flow can have different speed and direction with height

To a good approximation, many aspects of the atmosphere are equivalent barotropic

Rather than height and temperature, we can think of pressure and density



Baroclinic vs. Barotropic

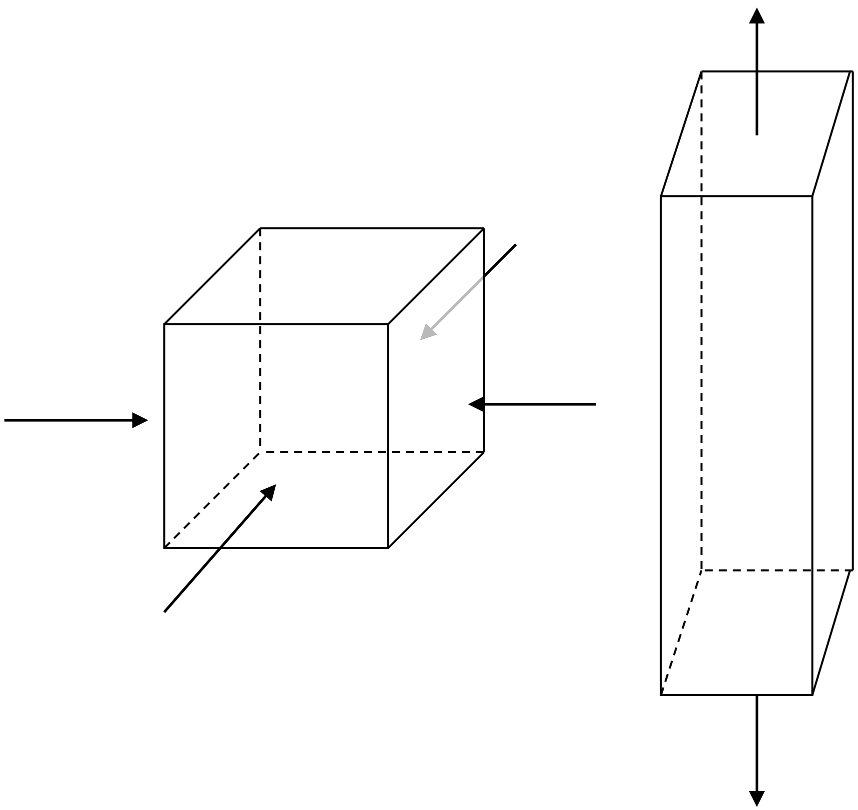
Barotropic	Baroclinic
ρ=ρ(p) only ($\nabla_p T = 0$)	ρ=ρ(p,T)
1) isobaric and isothermal surfaces coincide	1) isobaric and isothermal surfaces intersect
2) no vertical wind shear (thermal wind = 0)	2) vertical wind shear (thermal wind ≠ 0)
3) no tilt of pressure systems with height	3) pressure systems tilt with height

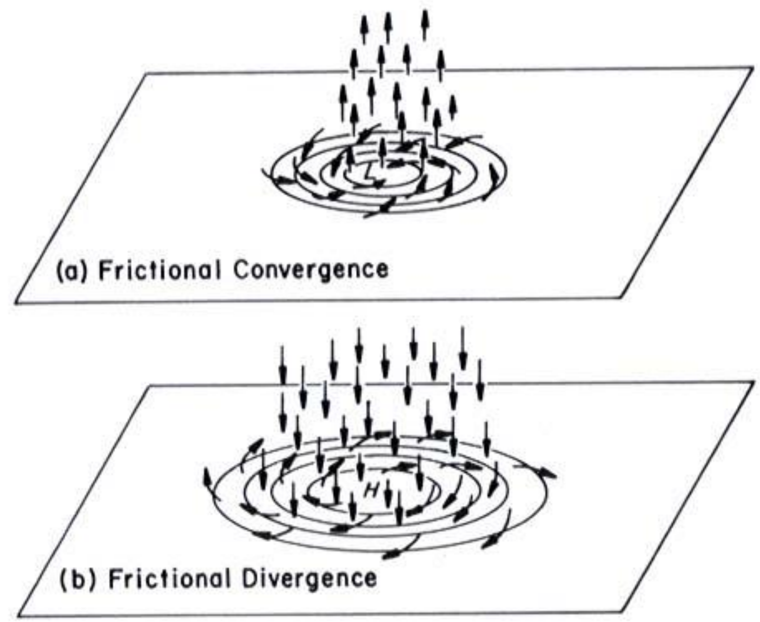
Seasons: Atmosphere is most baroclinic in winter.
Atmosphere is least baroclinic in summer.

Geographic: Atmosphere is most baroclinic in midlatitudes
Atmosphere is least baroclinic in the Tropics

Divergence (isobaric)

$$\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=-\frac{\partial \omega}{\partial p}$$





Typical Values for Horizontal Divergence

- Atmosphere: 10^{-5} to 10^{-6} s^{-1}
- Ocean: 10^{-7} to 10^{-8} s^{-1}

(cf. $f \sim 10^{-4}$ s^{-1})

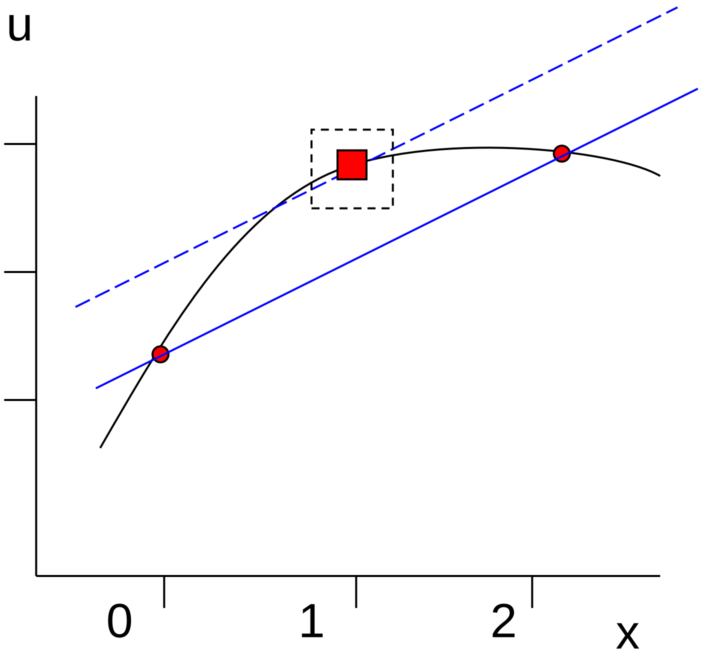
- For large-scale weather systems in midlatitudes, the horizontal velocity is close to geostrophic balance. If not for the small contribution due to the variation of the Coriolis parameter with latitude, the geostrophic wind would be nondivergent.
- Horizontal divergence in the atmosphere arises primarily due to the small departures of the wind from geostrophic balance.

Finite differences

- We can estimate derivatives for a region, where we can assume the properties of the fluid are nearly uniform

$$\frac{du}{dx} \approx \frac{\Delta u}{\Delta x} = \frac{u_b - u_a}{x_b - x_a}$$

$$\left(\frac{du}{dx}\right)_1 \approx \frac{u_2 - u_0}{x_2 - x_0}$$



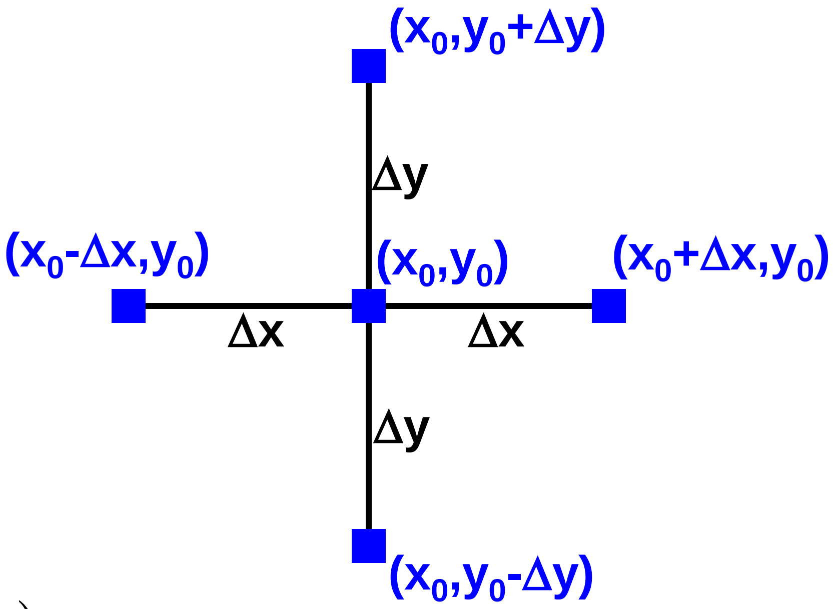
Finite differences very useful for solving differential equations with computers (e.g., the primitive equations)

Notice as the Δx goes to zero, this approximation becomes exact, as Newton told us

Estimating Horizontal Divergence

$$\nabla_H \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

horizontal divergence



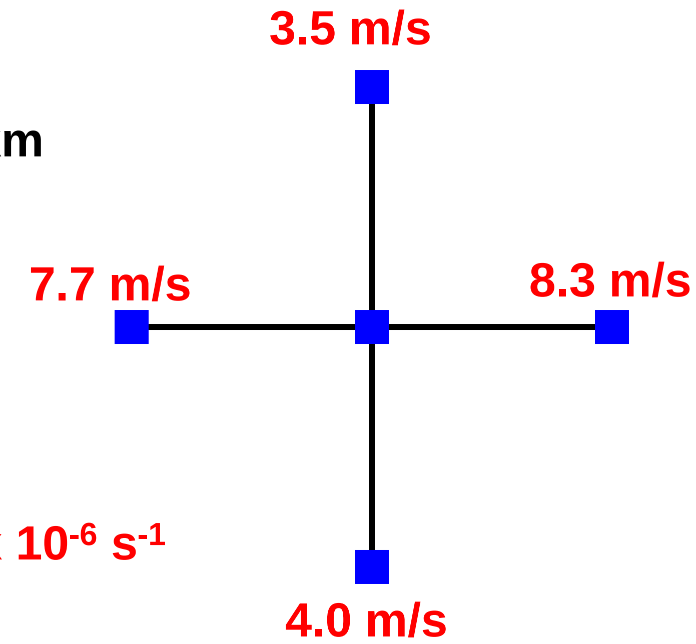
To compute horizontal divergence at (x_0,y_0), evaluate derivatives using centered finite differences:

$$\left(\frac{\partial u}{\partial x}\right)_{x_0,y_0} \approx \frac{u(x_0+\Delta x, y_0) - u(x_0-\Delta x, y_0)}{2\Delta x}$$

$$\left(\frac{\partial v}{\partial y}\right)_{x_0,y_0} \approx \frac{v(x_0, y_0+\Delta y) - v(x_0, y_0-\Delta y)}{2\Delta y}$$

Example:

$\Delta x = \Delta y = 50$ km



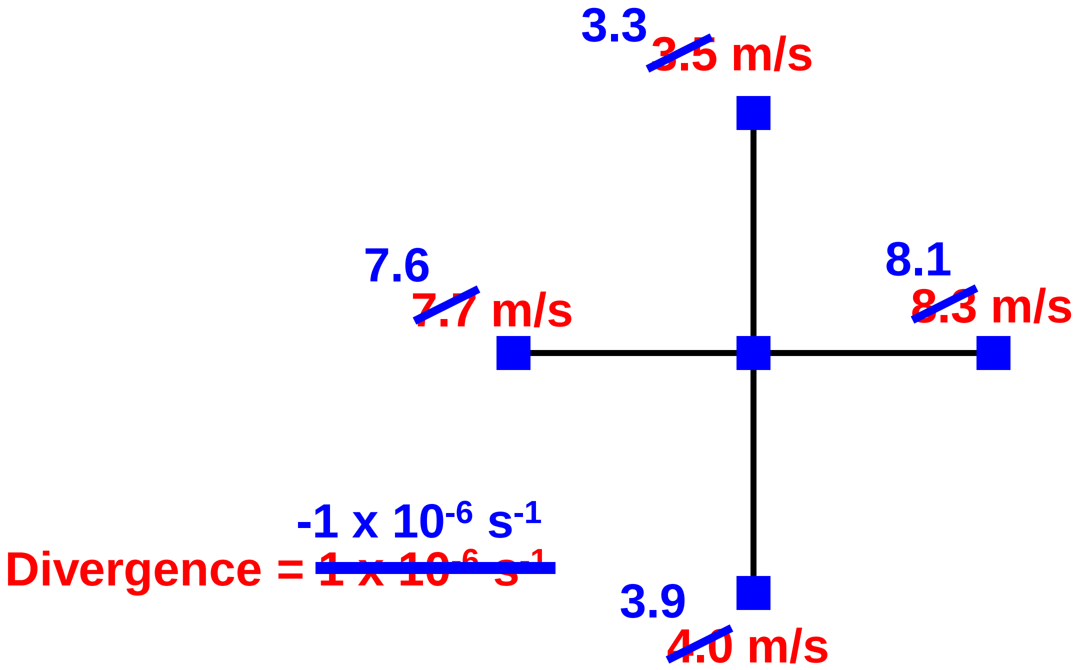
Divergence = 1 x 10^{-6} s^{-1}

$$\left(\frac{\partial u}{\partial x}\right)_{x_0,y_0} \approx \frac{8.3\,m\,s^{-1} - 7.7\,m\,s^{-1}}{2(50\,km)} = \frac{0.6\,m\,s^{-1}}{10^5\,m} = 6\times10^{-6}\,s^{-1}$$

$$\left(\frac{\partial v}{\partial y}\right)_{x_0,y_0} \approx \frac{3.5\,m\,s^{-1} - 4.0\,m\,s^{-1}}{2(50\,km)} = \frac{-0.5\,m\,s^{-1}}{10^5\,m} = -5\times10^{-6}\,s^{-1}$$

$$\nabla_2 \cdot \vec{V} = 6\times10^{-6}\,s^{-1} - 5\times10^{-6}\,s^{-1} = 1\times10^{-6}\,s^{-1}$$

Now, we introduce some small errors (0.1-0.2 m/s) in the wind estimates:



$$\left(\frac{\partial u}{\partial x}\right)_{x_0,y_0} \approx \frac{8.1\,m\,s^{-1} - 7.6\,m\,s^{-1}}{2(50\,km)} = \frac{0.5\,m\,s^{-1}}{10^5\,m} = 5\times 10^{-6}\,s^{-1}$$

$$\left(\frac{\partial v}{\partial y}\right)_{x_0,y_0} \approx \frac{3.3\,m\,s^{-1} - 3.9\,m\,s^{-1}}{2(50\,km)} = \frac{-0.6\,m\,s^{-1}}{10^5\,m} = -6\times 10^{-6}\,s^{-1}$$

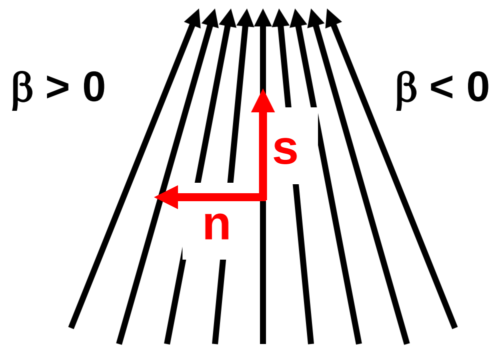
$$\nabla_2 \cdot \vec{V} = 5\times 10^{-6}\,s^{-1} - 6\times 10^{-6}\,s^{-1} = -1\times 10^{-6}\,s^{-1}$$

Divergence estimates using the kinematic method are very sensitive to small errors.

Divergence in Natural Coordinates

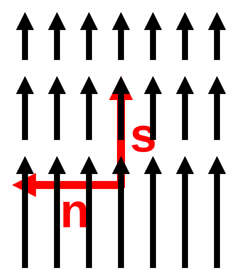
$$\nabla_H \cdot \vec{V} = -V\frac{\partial \beta}{\partial n} + \frac{\partial V}{\partial s}$$

β = angle between streamline and flow direction



$$-V\frac{\partial \beta}{\partial n} < 0$$

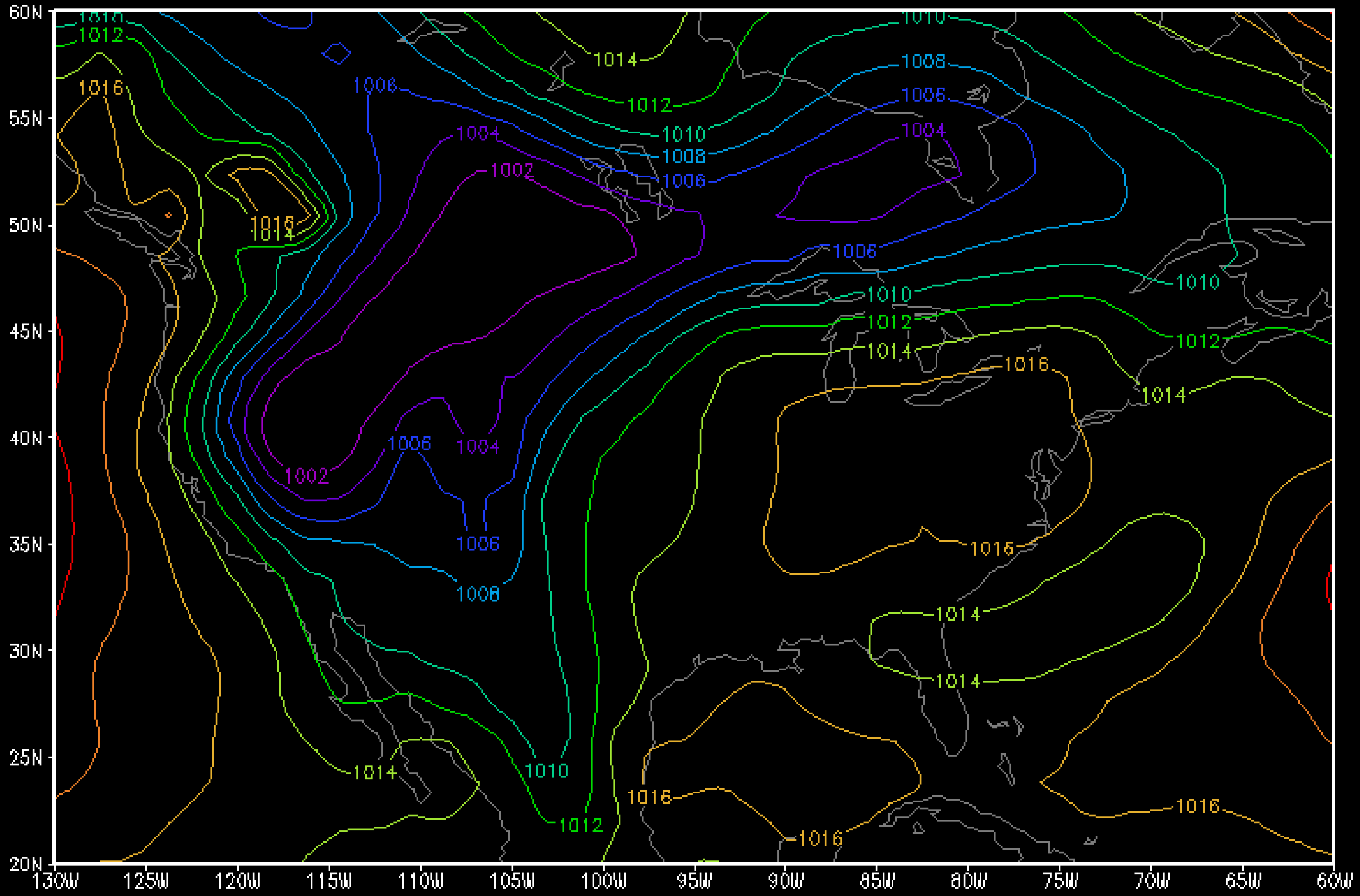
"directional convergence"



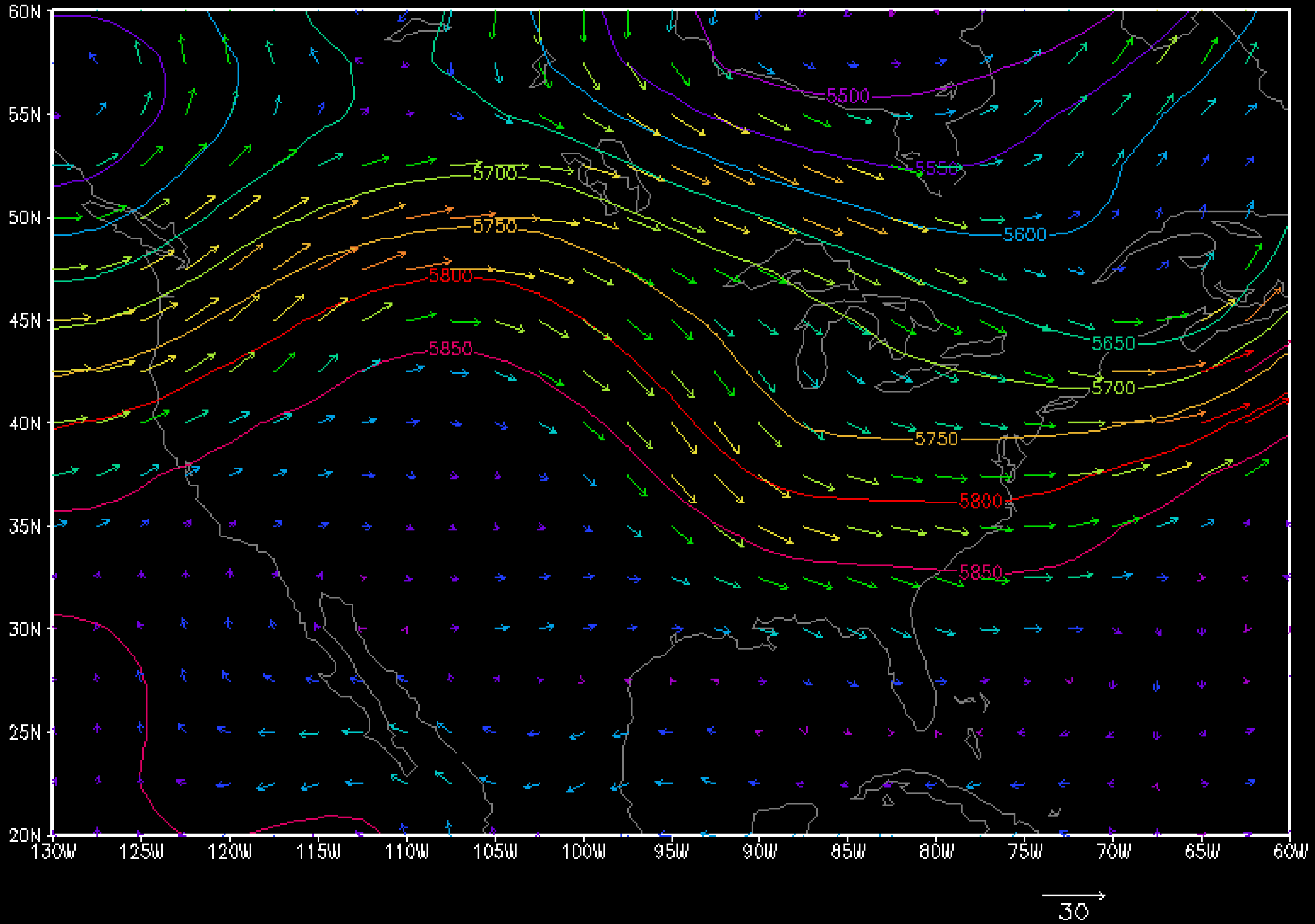
$$\frac{\partial V}{\partial s} < 0$$

"speed convergence"

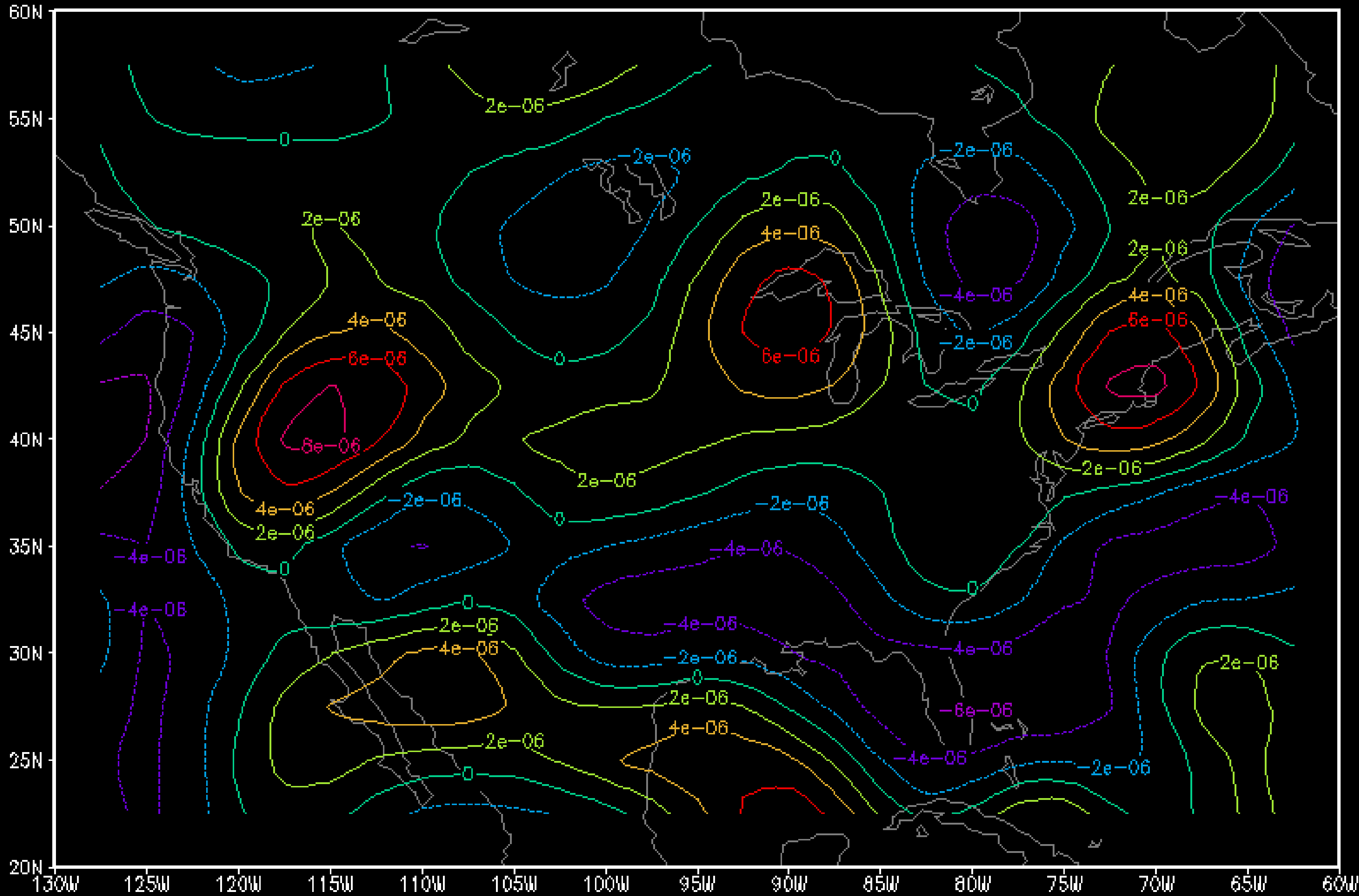
Mean sea-level pressure (mb) for 00Z01JUL2000



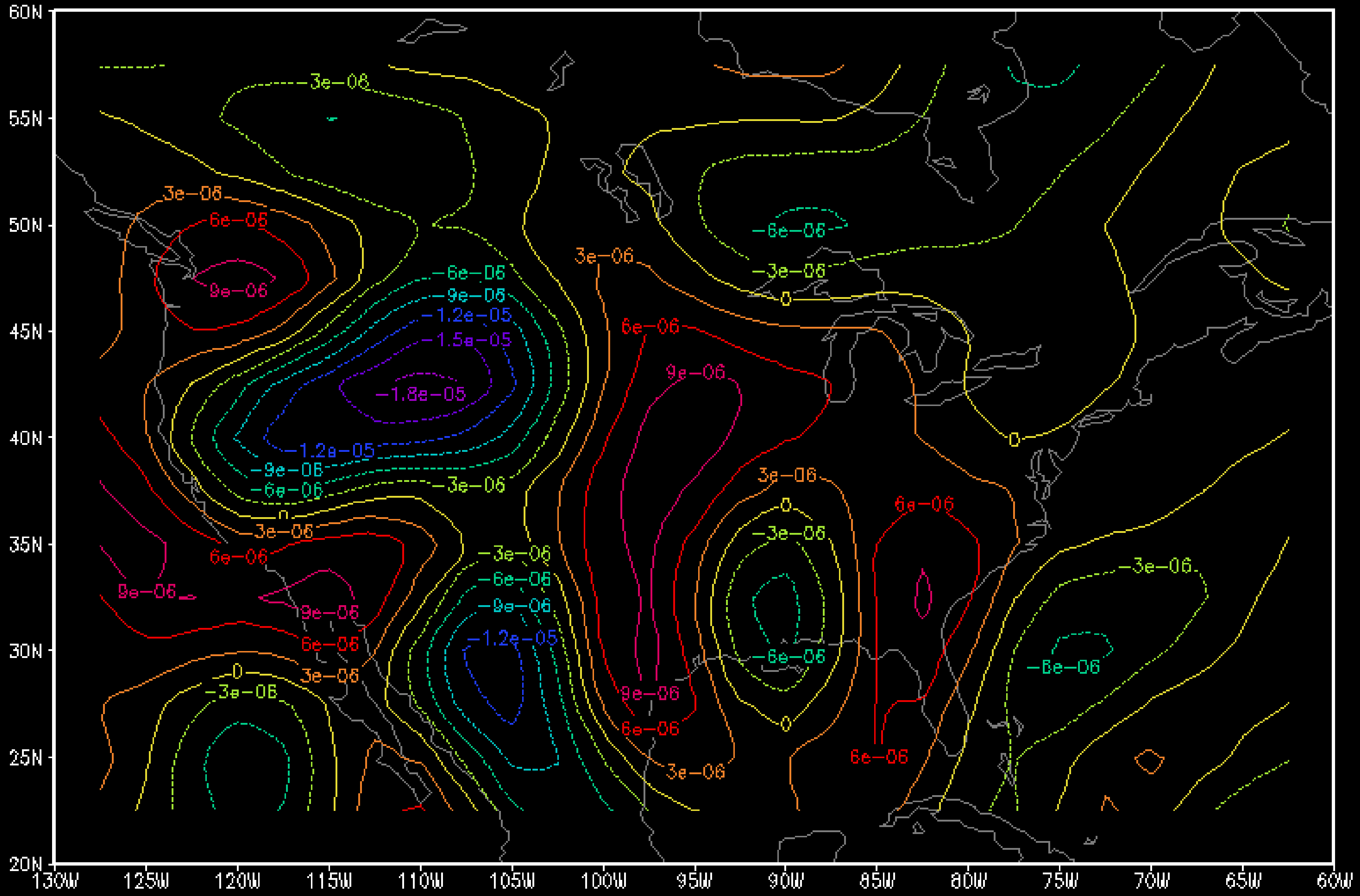
500 mb wind field and heights for 00Z01JUL2000



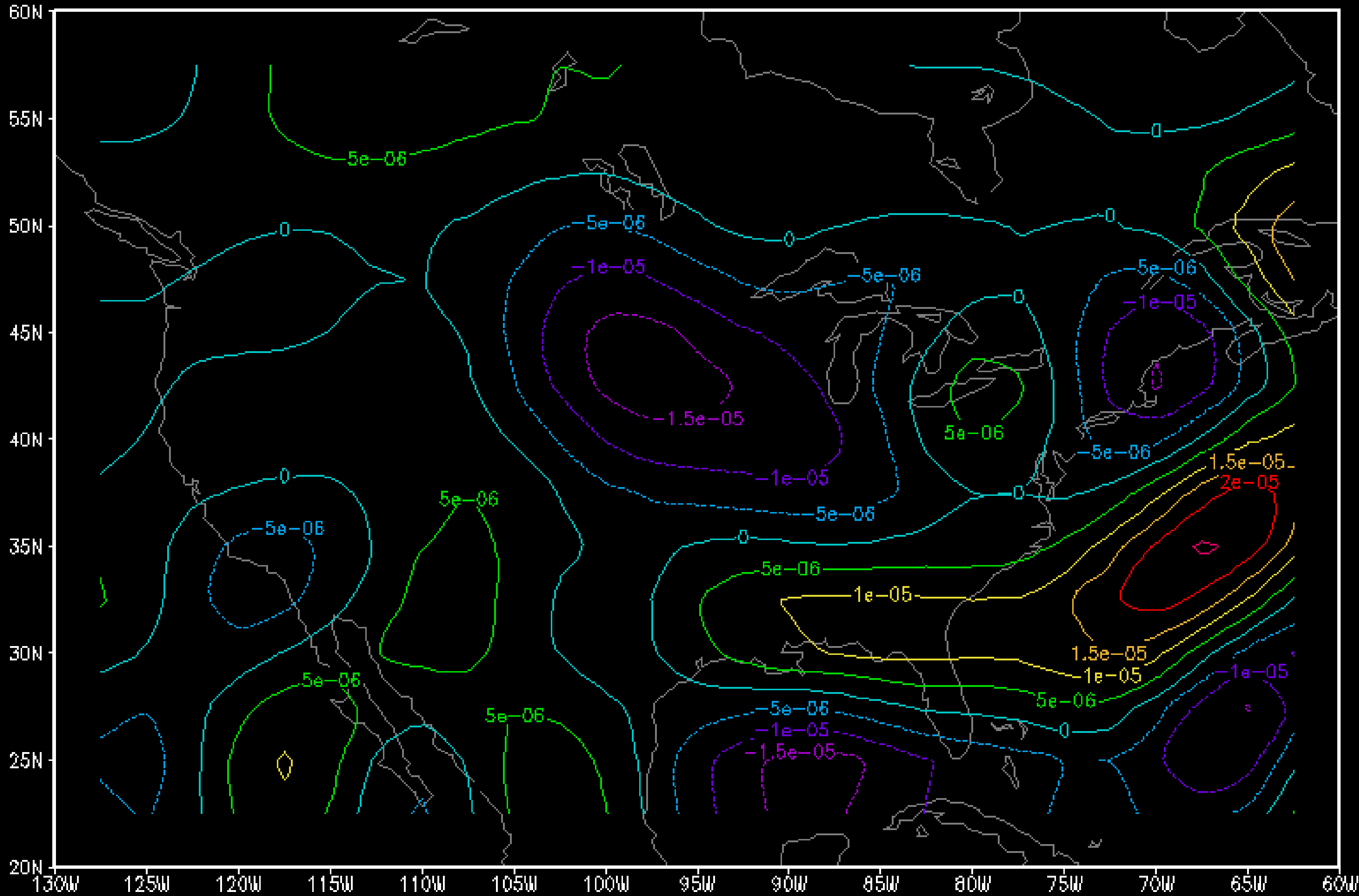
500 mb divergence (s^-1) for 00Z01JUL2000



850 mb divergence (s^-1) for 00Z01JUL2000



200 mb divergence (s^-1) for 00Z01JUL2000



Vertical motion

- Observations show vertical motions are small compared to horizontal motion (W~0.01 m/s, U ~10m/s)
- Scaling showed that vertical accelerations are tiny (the hydrostatic equation tells us the weight of the air is very close balanced by the vertical pressure gradient)
- Since vertical motion is balanced (hydrostatic), we can not predict changes in vertical velocity by integrating acceleration
- However, we can deduce the vertical velocity diagnostically, since we know vertical motion plays a role in continuity equation (divergence), and thermodynamic equation

Vertical velocity – kinematic method

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \omega}{\partial p}=0$$

$$d\omega=-\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dp$$

$$\omega(p)=\omega(p_s)+\left(p_s-p\right)\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$

We must know the horizontal divergence.

Accurately, if we want accurate vertical velocities

Recall large errors in divergence if small errors in velocity field

This is similar to saying we must be able to estimate the ageostrophic part of the flow, which is the order of the Rossby number smaller than the total flow field

(*Note: vertical velocity also appears in thermodynamic equation, so could use that also… knowing temperature advection and diabatic heating*)

Vertical velocity - thermodynamics

- Atmosphere mostly adiabatic, so thermodynamic equation:

$$\frac{dT}{dt} - \sigma\omega = \frac{J}{c_p} \approx 0$$

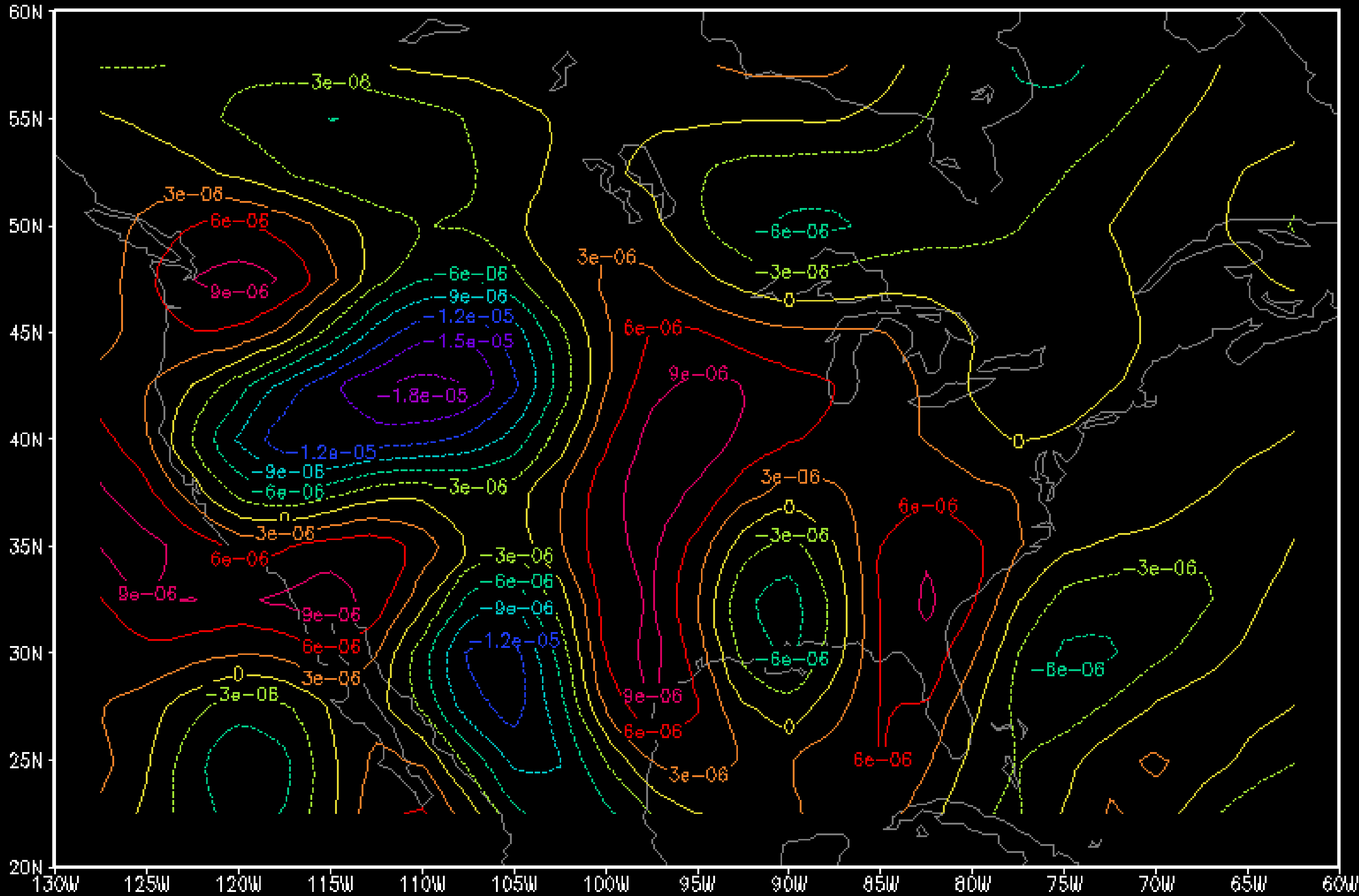
$$\sigma = -\frac{T}{\theta}\frac{\partial\theta}{\partial p}$$

$$\omega = \frac{1}{\sigma}\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right)$$

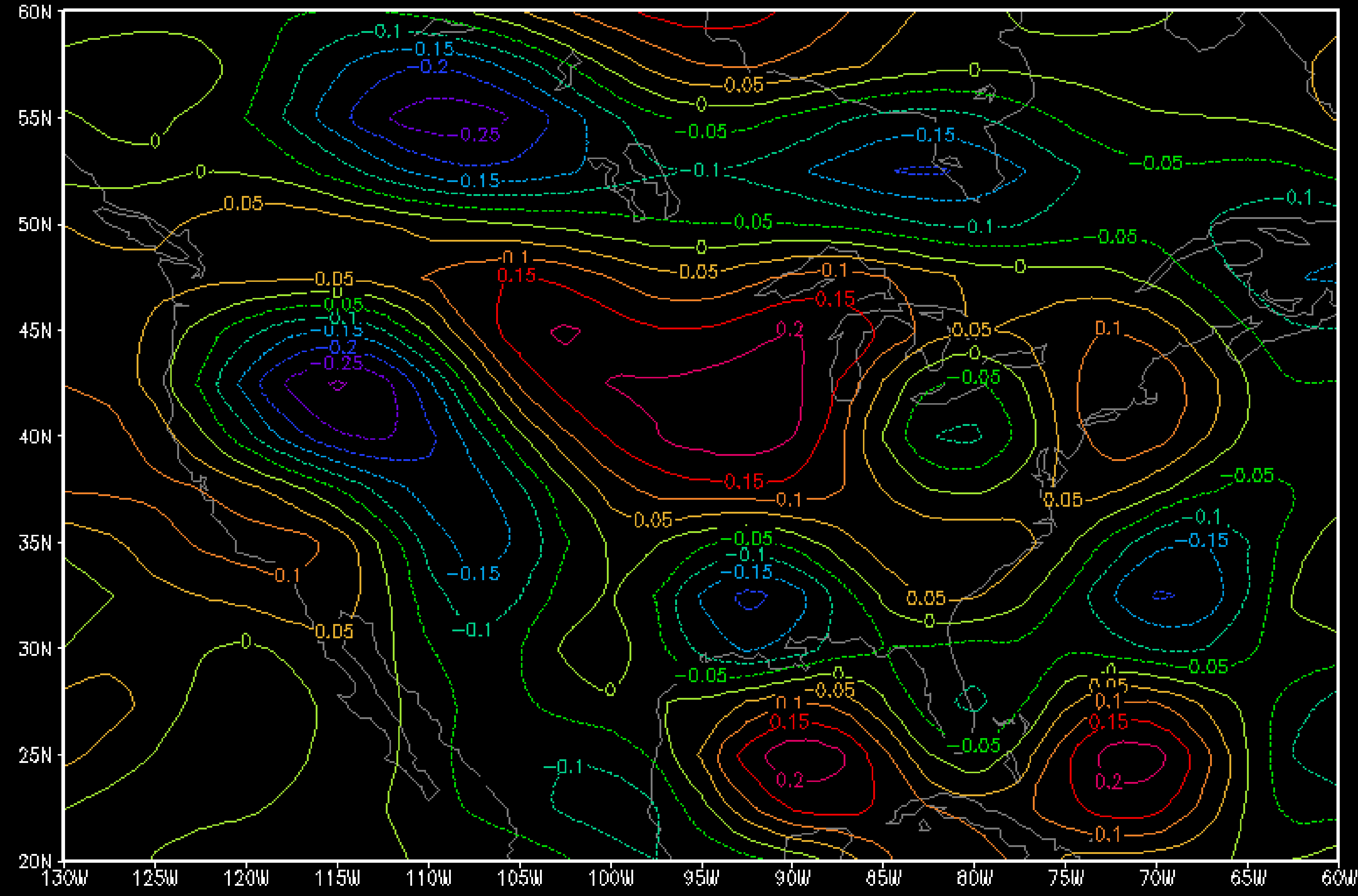
This form is much more robust, because temperature advection estimated from geostrophic assumption and thermal wind balance accurate to the order of the Rossby number.

This form accurate so long as we can assume diabatic heating is small

850 mb divergence (s^-1) for 00Z01JUL2000



500 mb vertical velocity (Pa/s) for 00Z01JUL2000



Pressure tendency

Divergence a measure of air mass movement way from a location

Since pressure is a measure of mass, can use vertical integral of divergence to estimate surface pressure changes

$$d\omega = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)dp$$

$$\frac{\partial p_s}{\partial t} \approx -\int_0^{p_s} \left(\nabla \cdot \mathbf{V}\right)dp$$

This is a consequence of hydrostatic balance (pressure and mass related).

Divergence and geostrophy

- If the flow is non-divergent in the horizontal, there is no vertical motion
- So ageostrophic conditions lead to vertical motions
- Recall that the acceleration of the flow is due to ageostrophic component of the flow.

$$\frac{1}{\rho}\frac{\partial p}{\partial x} = fv_g \qquad \frac{1}{\rho}\frac{\partial p}{\partial y} = -fu_g$$

$$\frac{du}{dt} = f\left(v - v_g\right) = fv_a \qquad \frac{dv}{dt} = -f\left(u - u_g\right) = -fu_a$$

So can only have vertical motion where there is acceleration (net force imbalance)

This has important implications for development of weather systems

Weekly exercise

Holton 3.20, 3.21, 3:22

Derive an expression for the divergence of the geostrophic flow, and compute a typical value for large scale midlatitude flow.

Other practice problems:

- Holton: 3.1 – 3.10 are all good review
- Holton: 3.18-3.23