Balanced flow

Force balance

- Pressure gradient force: $$\nabla\Phi = \mathbf{t}\frac{\partial\Phi}{\partial s} + \mathbf{n}\frac{\partial\Phi}{\partial n}$$
- Coriolis force (normal to flow): $-fV\mathbf{n}$
- Thus tangent and normal components of momentum equation

$$\frac{dV}{dt} = -\frac{\partial\Phi}{\partial s} \qquad \frac{V^2}{R} = -fV - \frac{\partial\Phi}{\partial n}$$

Change in speed due to pressure gradient in direction of flow

Thus balanced flow ($d\mathbf{V}/dt$ = 0) must be at constant height
(flow follows contours of geopotential height)

If latitudinal variations in *f* can be neglected, constant geopotential gradient normal to flow implies constant *R*
(flow is circular)

Gradient wind balance

$$\frac{V^2}{R} = -fV - \frac{\partial \Phi}{\partial n}$$

Balanced flow (no friction)
Balance = constant speed

No change in pressure following trajectory. So all isobaric.

Solving for wind speed:

$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n}}$$

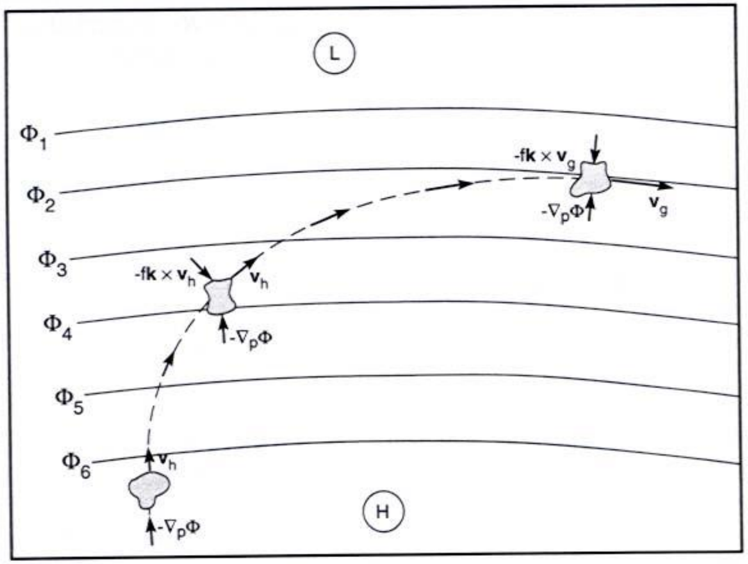
V defined as positive, and must be real, thus four possible flow configurations

Geostrophic flow

- Balance of pressure gradient and Coriolis forces

$$\frac{V^2}{R} = -fV - \frac{\partial \Phi}{\partial n}$$

$$fV = -\frac{\partial \Phi}{\partial n}$$



Strictly, satisfied only as curvature, *R*, becomes infinite ($V^2 << R$) i.e. flow is straight line

However, a good approximation for most large scale flows (to the order of the Rossby number)

$$V_g = -\frac{1}{f}\frac{\partial \Phi}{\partial n}$$

How long for a parcel to make one circle?
$P = |2\pi R/V|$

Inertial flow

Absence of geopotential gradient,

$$\frac{V^2}{R} = -fV - \frac{\partial \Phi}{\partial n}$$

$$R = -\frac{V}{f}$$

This is quite rare in nature, although can occur in oceans where flow is not initiated by geopotential gradients

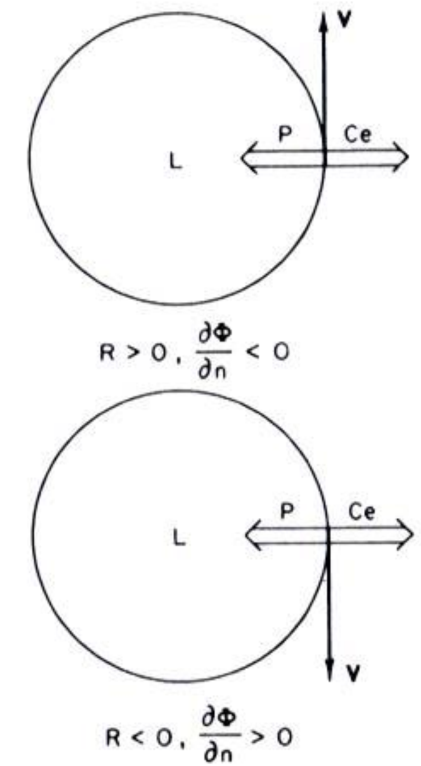
Conservation of angular momentum, with centripetal force

Since *V* defined positive, *R* < 0 in NH and R > 0 in SH.
Both cases flow is anticyclonic

Cyclostrophic flow

For flow with Rossby number >1, can ignore effects of rotation
i.e., fast or small – recall $R_o = U/(f_0L)$

$$\frac{V^2}{R} = -\cancel{fV} - \frac{\partial \Phi}{\partial n}$$



V defined positive, so *R* has same opposite sign as geopotential gradient (may be positive or negative)

Thus flow may be either cyclonic or anticyclonic,
as dictated by the flow in which the system is embedded

Cyclostrophic wind speed:

Undefined if forces no opposed.

$$V = \sqrt{-R\frac{\partial \Phi}{\partial n}}$$

e.g., tornadoes, drains/toilets, …

Gradient wind balance

$$\frac{V^2}{R} = -fV - \frac{\partial \Phi}{\partial n}$$

Balanced flow (no friction)

More complicated (3- way balance), however, better approximation than geostrophic (as allows for centrifugal acceleration due to curvature of parcel trajectory)

Solving for wind speed (plug and chug in the quadratic formula):

$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2 R^2}{4} - R\frac{\partial \Phi}{\partial n}}$$

V defined as positive, and must be real, thus four possible flow configurations

Gradient wind solutions

$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}}$$

$$fV_g = -\frac{\partial\Phi}{\partial n}$$

V real if $\left|\frac{f^2R^2}{4}\right| > \left|R\frac{\partial\Phi}{\partial n}\right|$

V positive if

$$\frac{fR}{2} < +\sqrt{\frac{f^2R^2}{4} - R\frac{\partial\Phi}{\partial n}}$$

$$\frac{fR}{2} < +\sqrt{\frac{f^2R^2}{4} + fRV_g}$$

R >0 or R < 0

Thus 4 possible physical configurations:

R > 0, V_g> 0 normal low

R < 0, V_g> 0 normal high ($V > -fR/2$)

R < 0, V_g> 0 anomalous high ($V < -fR/2$)

R < 0, V_g< 0 anomalous low ($V_g < 0$!)

Unphysical solutions if all forces in same direction.

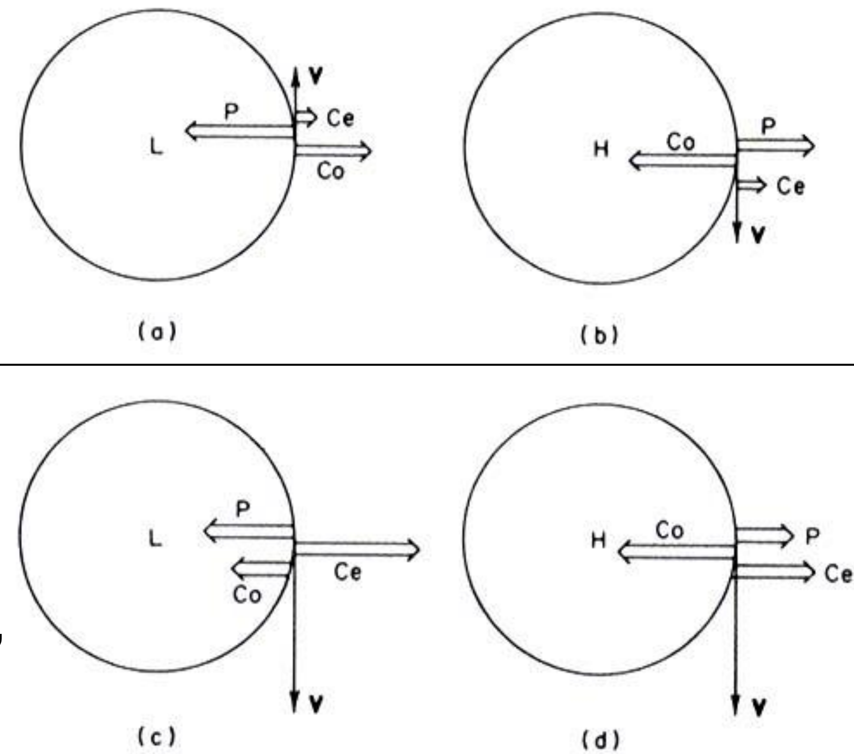
R < 0 is cyclonic, R> 0 anticyclonic

Possible gradient flow

regular

anomalous

Unstable as large centrifugal required, rarely sustained



Can occur near tropics (regular high crossing equator)

Fig. 3.5 Force balances in the Northern Hemisphere for the four types of gradient flow: (a) regular low (b) regular high (c) anomalous low (d) anomalous high.

Notice centrifugal force always “out”

Strength of highs and lows

$$V = -\frac{fR}{2} \pm \sqrt{\frac{f^2R^2}{4} + fV_g}$$

$$V = -\frac{fR}{2} \pm \frac{fR}{2}\sqrt{1 + \frac{4V_g}{fR}}$$

For an anticyclone, R < 0, real roots require

$$\frac{4V_g}{f\left|R\right|} < 1 \qquad V_g < \frac{Rf}{4}$$

- That is, a maximum wind speed allowed ($V = 2\ V_g$)!
- For a cyclone there is no such limit
  (as root is always positive for R > 0)

So, low pressure systems can be deeper than high pressure systems can be high.

- Highs tend to be wide and flat features
  Lows tend to be compact circular features

Comparison of balanced flows

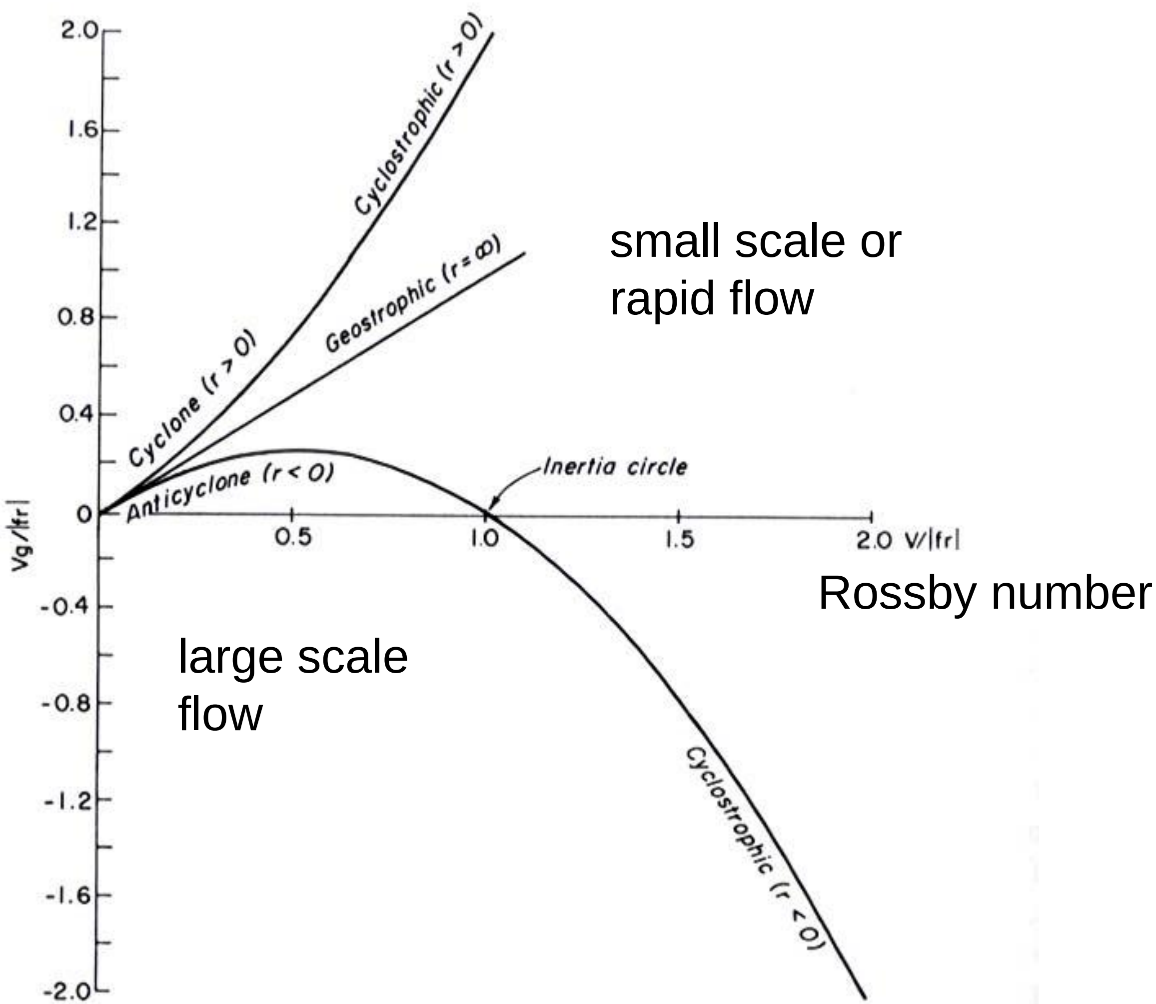


FIG. 4.8. Gradient wind diagram for circular flow in the counterclockwise direction ($r > 0$) and the clockwise direction ($r < 0$) in the Northern Hemisphere.

Fleagle and Businger, An introduction to Atmospheric Physics,

Gradient versus geostrophic

$$\frac{V^2}{R} = -fV - \frac{\partial \Phi}{\partial n} \qquad fV_g = -\frac{\partial \Phi}{\partial n}$$

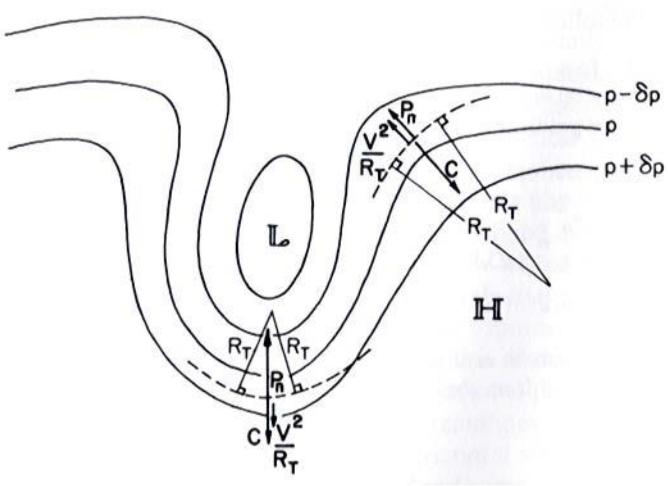
$$\frac{V^2}{R} = -fV - fV_g$$

$$\frac{V_g}{V} = 1 + \frac{V}{fR}$$

- Differ in magnitude by the Rossby number (Ro= U/fL = V/fR)
- Geostrophic flow stronger for cyclonic flow ($fR > 0$)
- Geostrophic flow is weaker for anticyclonic flow ($fR < 0$)
- Recall from scaling arguments that Ro ~ 0.1 for large scale motions

Geostrophic wind is within about 10% of gradient wind

Gradient wind



$$\frac{V^2}{R} = -fV - \frac{\partial \Phi}{\partial n}$$

V has 4 possible physical solutions:

R > 0, V_g > 0 normal low

R < 0, V_g > 0 normal high ($V > -fR/2$)

R < 0, V_g > 0 anomalous high ($V < -fR/2$)

R < 0, V_g < 0 anomalous low ($V_g < 0$!)

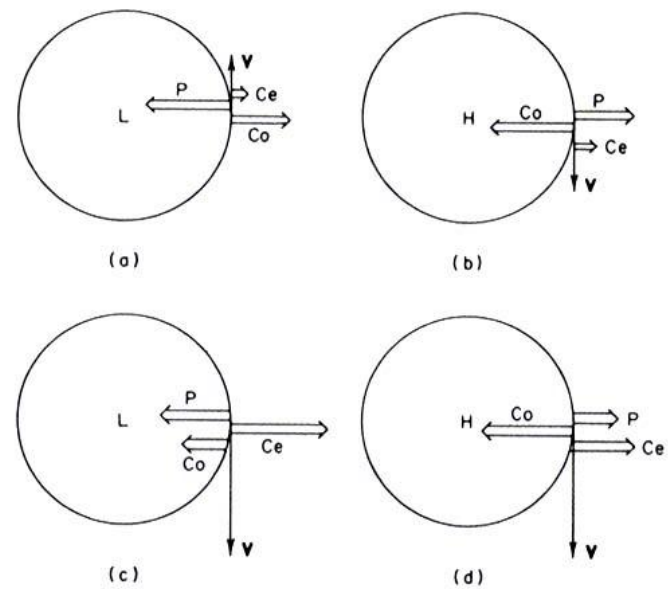
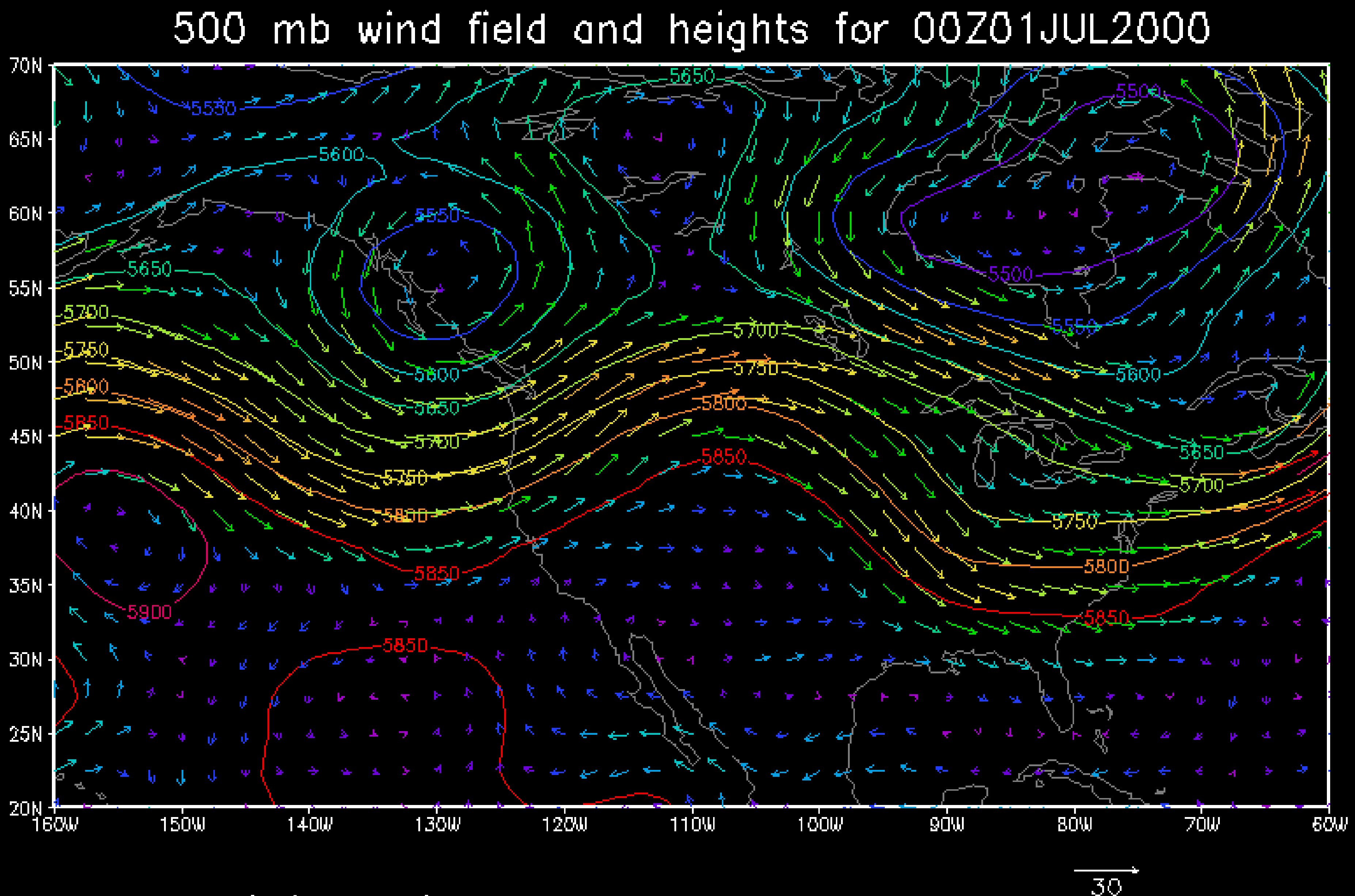
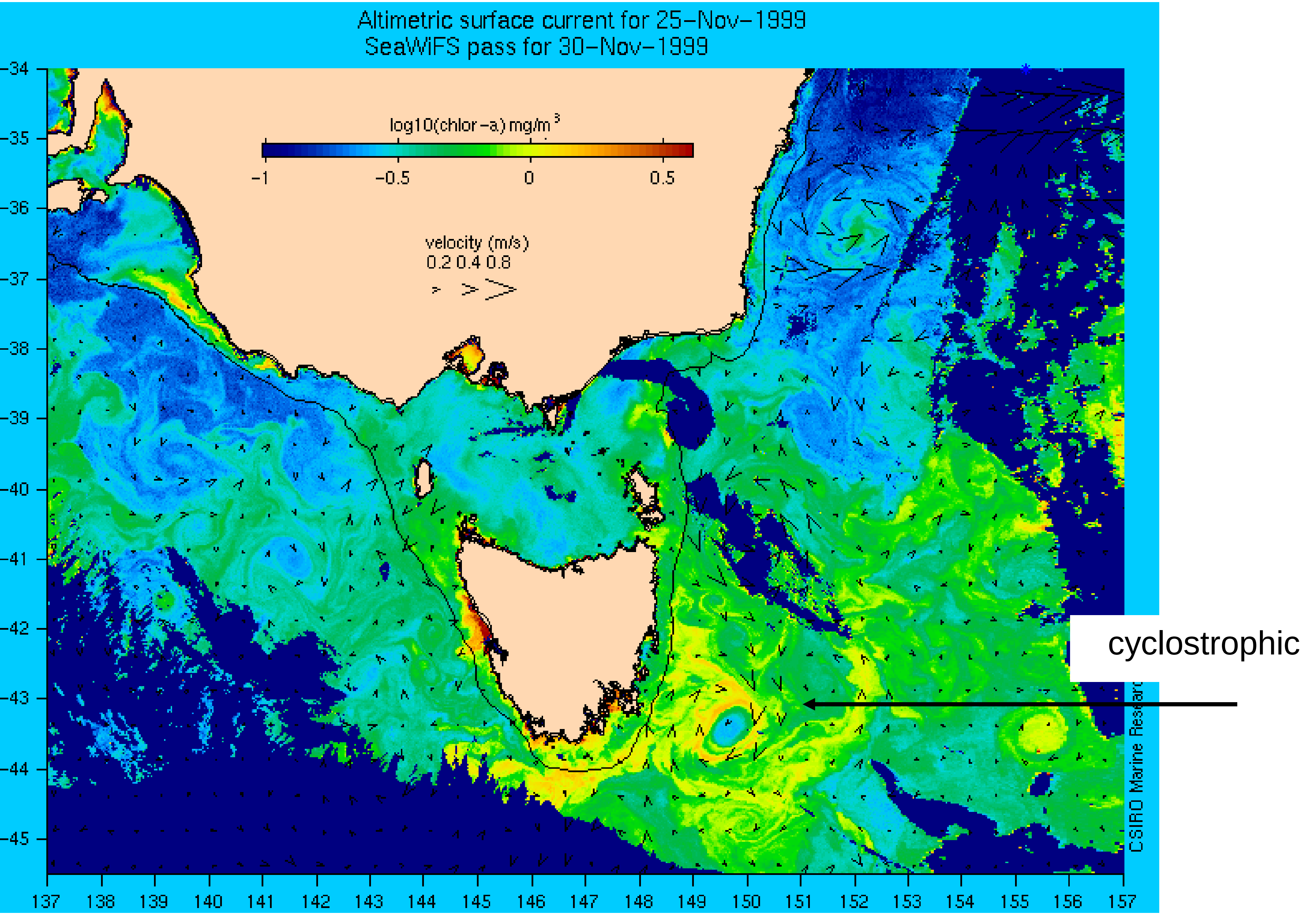


Fig. 3.5 Force balances in the Northern Hemisphere for the four types of gradient flow: (a) regular low (b) regular high (c) anomalous low (d) anomalous high.

Large scale flow almost geostrophic



NCEP Reanalysis – 1 July 2000



Hurricanes also almost cyclostrophic