

Moisture

# Euler's equation (reminder)

$$e^{ix} = \cos x + i \sin x$$

“the most beautiful, profound and subtle expression in mathematics” - Bertrand Russell

“the most amazing equation in all of mathematics” -  
Richard Feynman

- This is the basis for Fourier analysis, or, more generally, ways of describing waves
- So waves in the form (for frequency  $w$ , and amplitude  $A$ )

$$z(t) = Ae^{iwt}$$

# Example: buoyancy waves

- Consider a (laminar) flow over a mountain in a stable environment
- The mountain provides a mechanical displacement
- The positive restoring force accelerates the air mass toward the initial altitude
- The parcels overshoot, and are now displaced downward.
- Again the positive restoring forces accelerates the parcel toward the initial location.... And so on
- Friction ultimately reduces the amplitude

# Buoyancy waves

$$\frac{d^2 z}{dt^2} + \frac{g}{T} (\Gamma - \Gamma') z = 0$$

$$\frac{d^2 z}{dt^2} + N^2 z = 0$$

- This is the form of a simple oscillator, which has a solution

$$z(t) = A e^{iNt}$$

- Define  $N$ , the Brunt-Väisälä (buoyancy) frequency

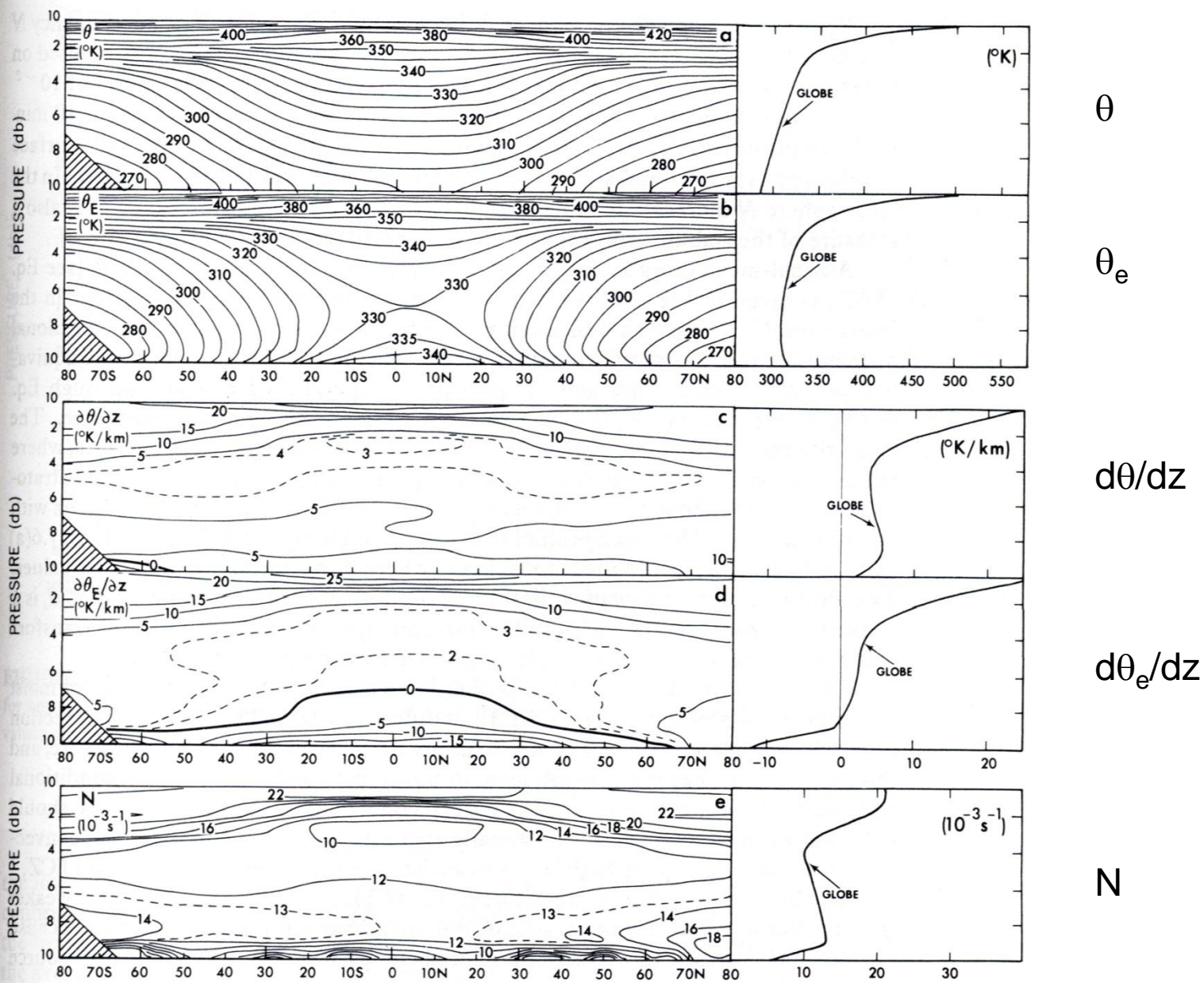
$$N^2 = \frac{g}{T} (\Gamma_d - \Gamma) = g \frac{d \ln \theta}{dz}$$

$N$  can be considered a measure of the “stiffness” of the atmosphere (think of the spring equation) ( $\sim 12 \times 10^{-2}$  /s locally,  $1 \times 10^{-3}$  large-scale)

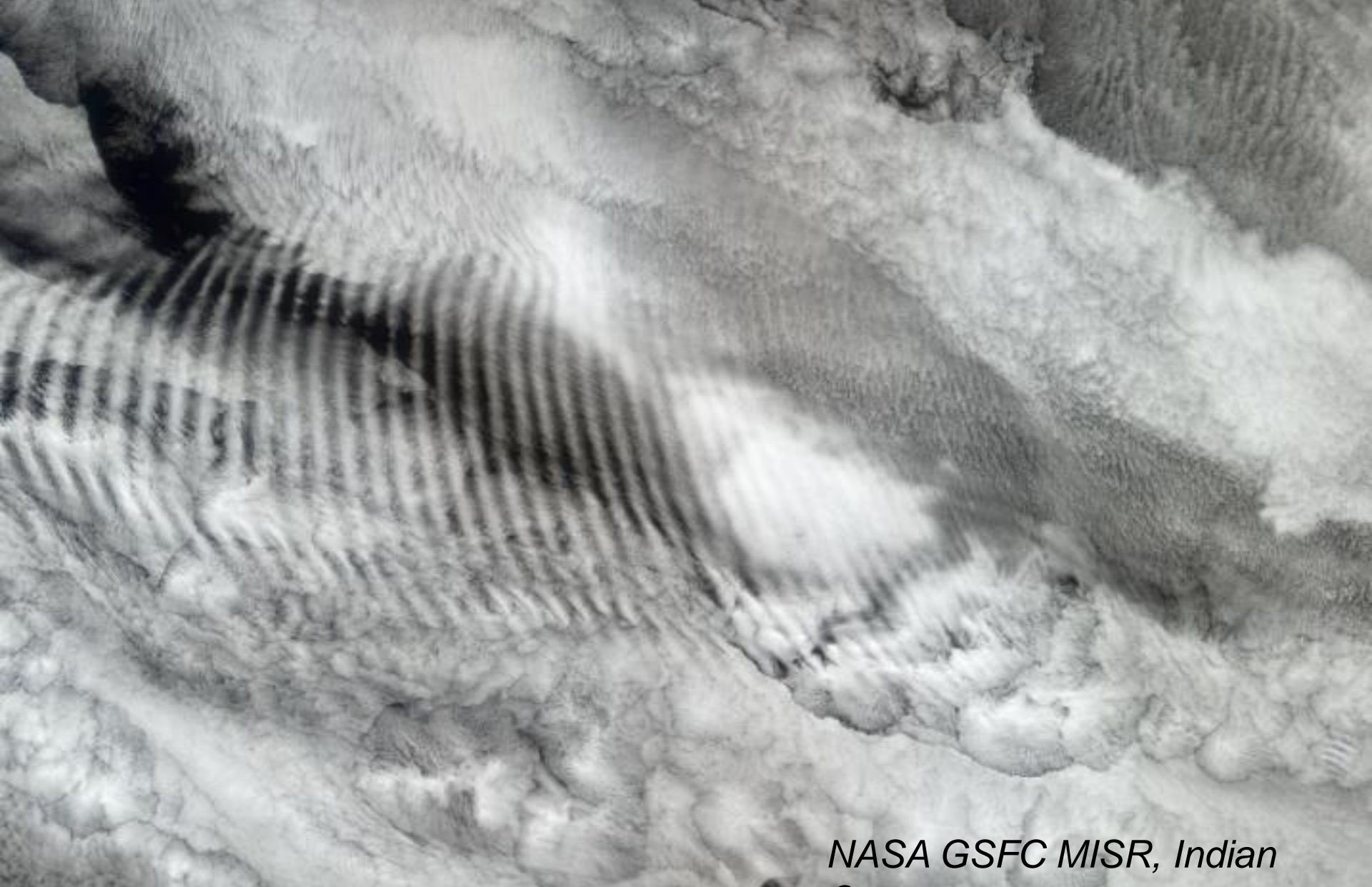
The period of the oscillation is simply  $T = 2\pi/N$  ( $\sim 1$ -10 minutes)

# Unstable growth

- If  $N^2 < 0$ ,  $N$  is imaginary, so our solution becomes,  
$$z(t) = Ae^{N't} \quad (\text{where } N'^2 = -N^2 > 0)$$
- This does not satisfy Euler's equation and indeed describes exponential growth of the displacement
- In practice this is growth is reduced by friction, but explains deep convection in the tropics in which parcels ascend until they reach the tropopause
- The stratosphere is very stable and prohibits any continual instability



**FIGURE 7.6.** Zonal-mean cross sections of the potential temperature (a) in K, equivalent potential temperature (b) in K, vertical gradient of potential temperature (c) in K/km, vertical gradient of equivalent potential temperature (d) in K/km, and Brunt-Väisälä frequency in  $10^{-3} \text{ rad s}^{-1}$  for annual-mean conditions. Vertical profiles of the global mean values are shown on the right.



*NASA GSFC MISR, Indian  
Ocean*

Which direction is background flow?  
Which direction do mountain waves move?



# Moisture

- Energy is stored during evaporation
- Energy is released during condensation
- This latent heat release is important for driving atmospheric motions (diabatic heating, future class)
- Condensation changes thermodynamic structure (e.g., thunderstorms)
- Variations in water vapor also change mass of “air”

# Moisture variables

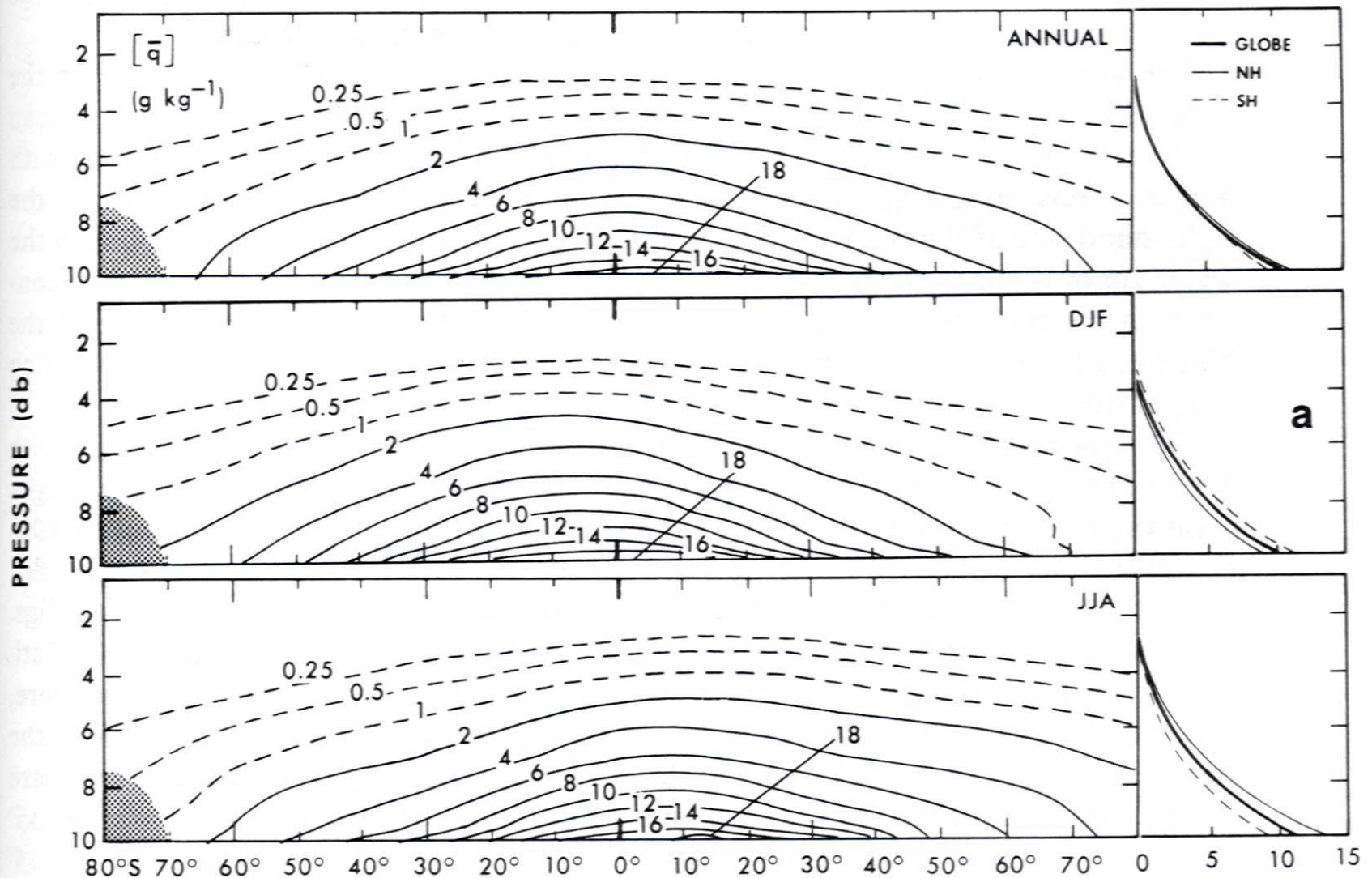
- Atmosphere is dry air plus water vapor
- New state variable to describe water vapor  
Water vapor also an ideal gas ( $R_v$ , for vapor)
- Partial pressure,  $e$ ,  $e = \rho_v R_v T$
- So total pressure  $p = p_d + e$

1. Vapor pressure:  $e$  [Pa]
  2. Specific humidity:  $q = m_v / m = m_v / (m_d + m_v)$  [kg/kg]
  3. Mixing ratio:  $w = m_v / m_d \sim q$  [kg/kg]
  4. Relative humidity:  $h = e / e_s$  [%]
- Others, e.g., dew point temperature...

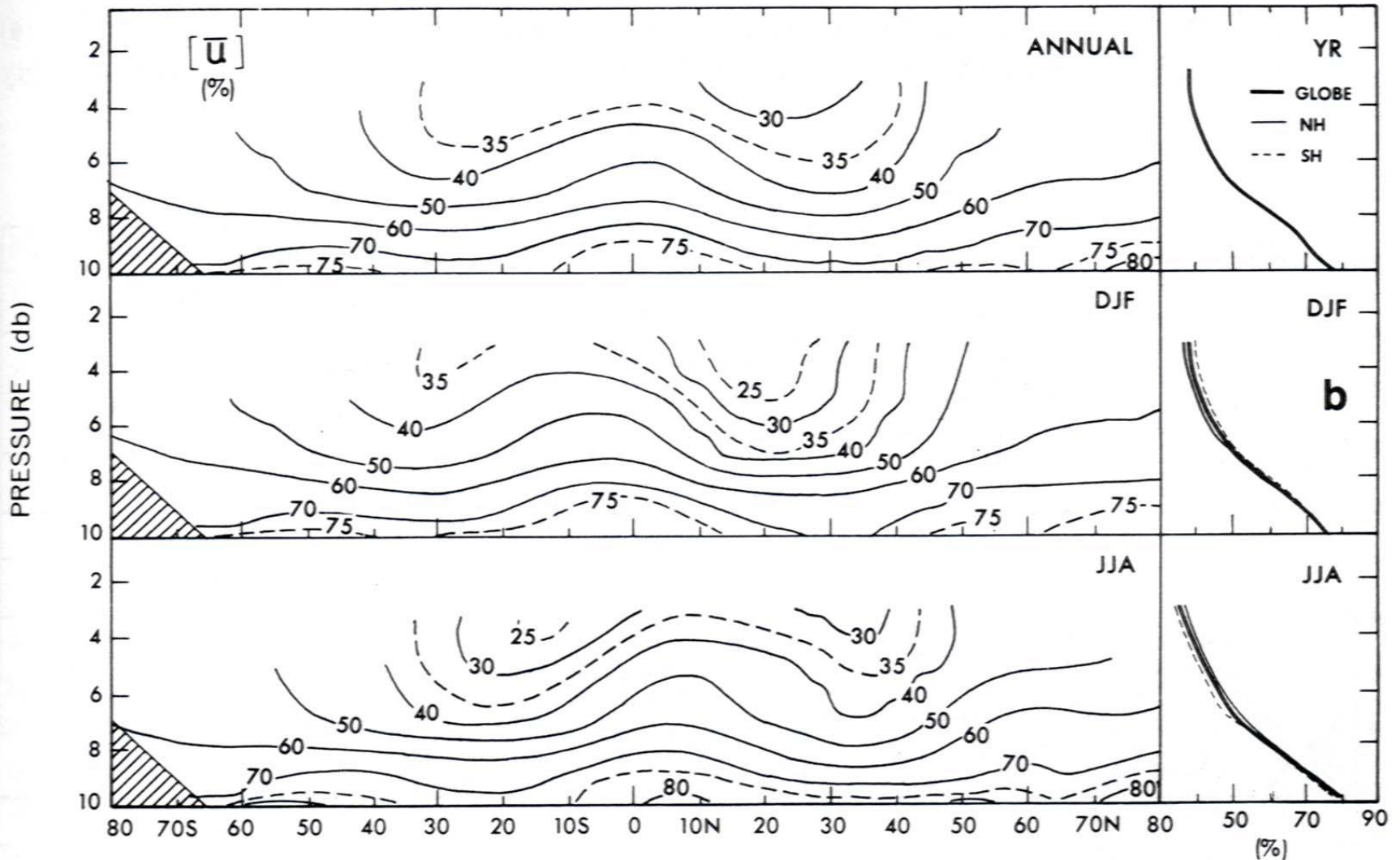
Each has various advantages and shortcomings

All are related to one another (as per homework!), and only one additional state variable

# Zonal mean specific humidity



# Zonal mean relative humidity



# Moisture effect on density

- Molecular weight of air is about 29 g/mol
- Molecular weight of water vapor is 18 g/mol
- So changes in vapor content modifies density.
- It is convenient to absorb this effect into a state variable
- If the system is unsaturated, we define virtual temperature  $T_v$

$$p = p_d + e = \rho_d R_d T + \rho_v R_v T$$

$$R = (1 - q)R_d + qR_v$$

$$R = (1 - q)R_d + \frac{q}{\varepsilon} R_d$$

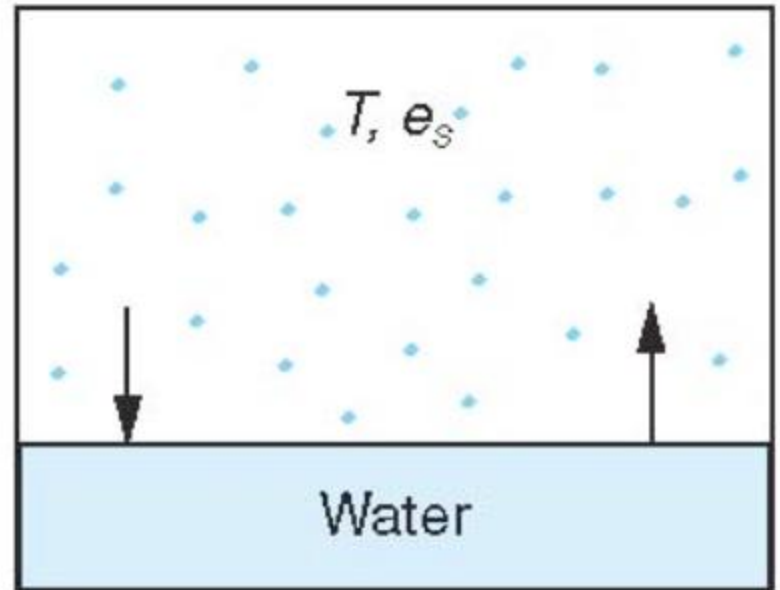
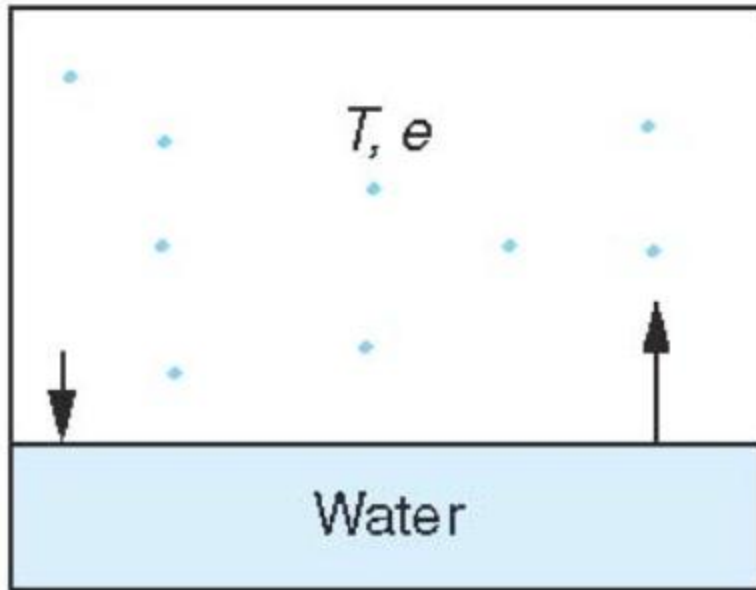
$$R = \left[ 1 + \left( \frac{1}{\varepsilon} - 1 \right) q \right] R_d$$

$$R = (1 + 0.61q)R_d$$

$$T_v = (1 + 0.61q)T$$

Thus, the equation of state,  $p = \rho R_d T_v$

# Saturation

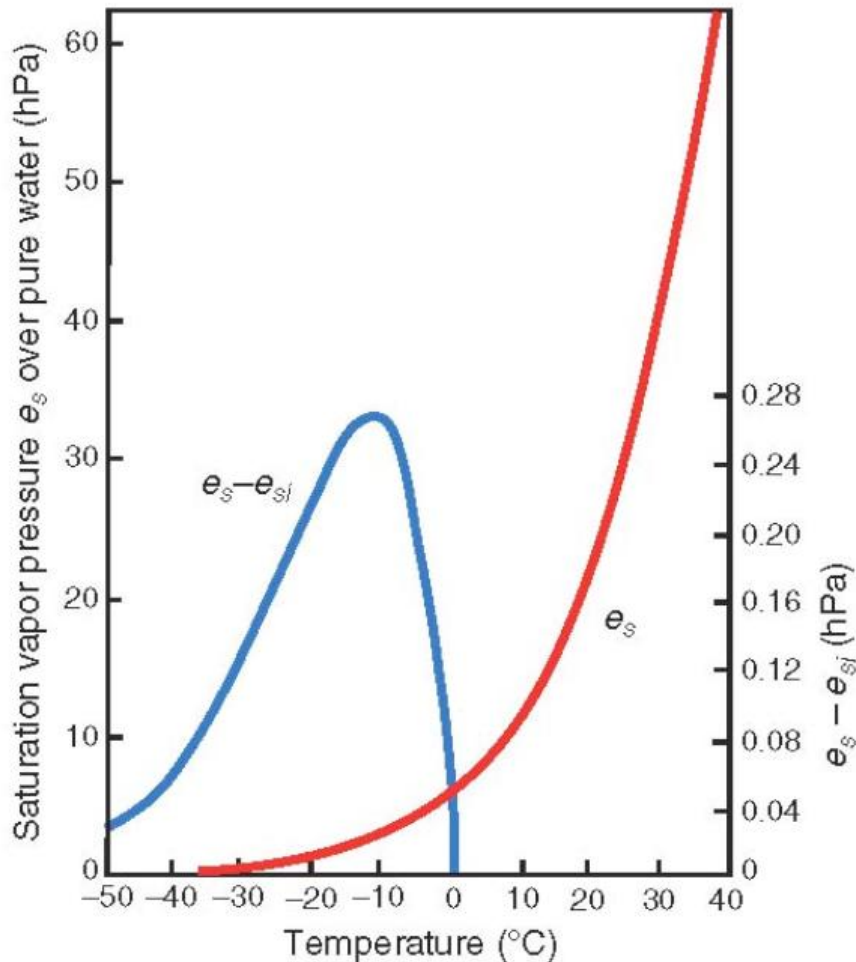


- Molecules jumping out of the liquid equals number jumping back in.
- Former larger, called evaporation
- Latter larger, called condensation

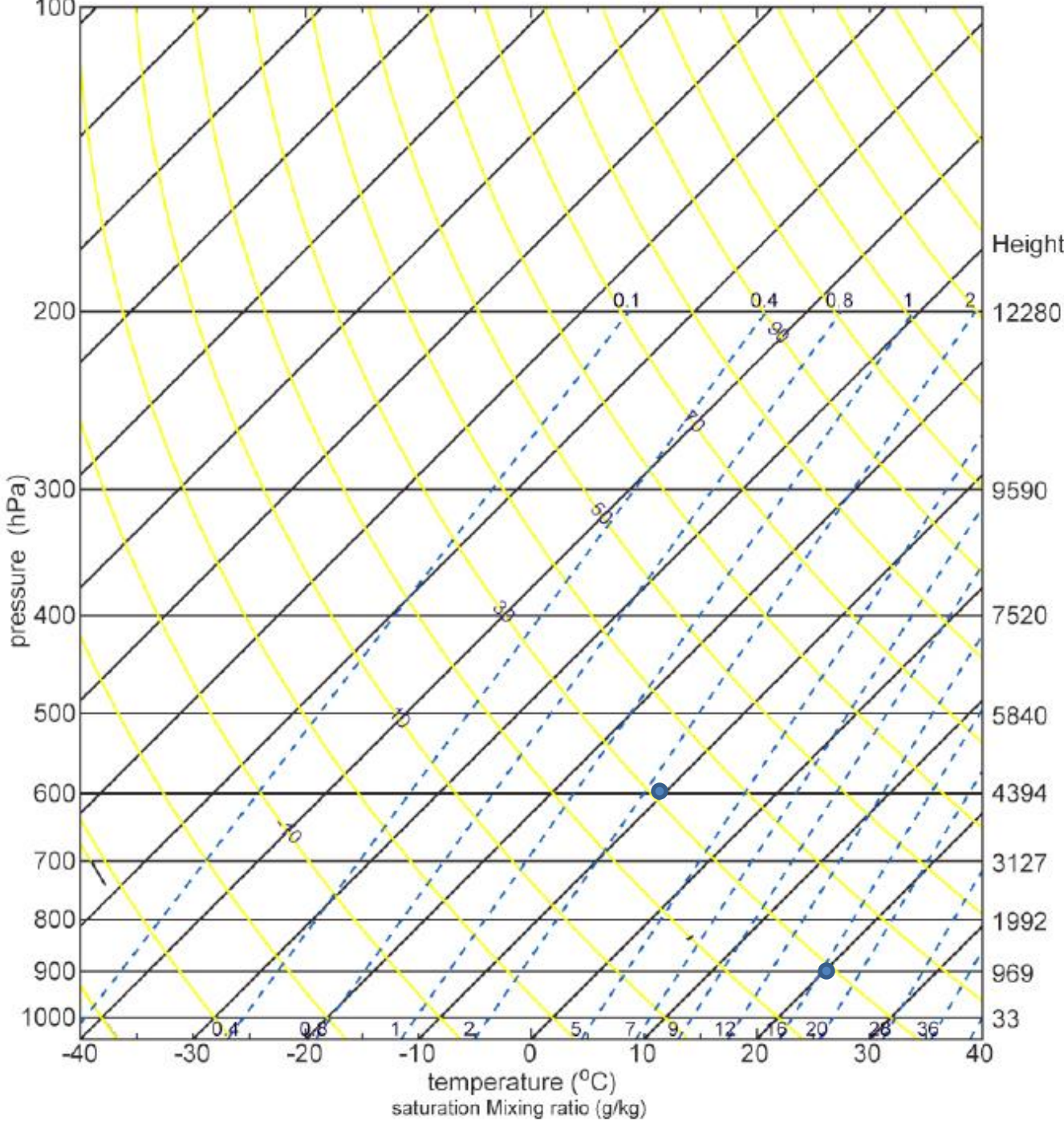
Saturation vapor pressure ( $e_s$ ): *The partial pressure that would be exerted by water vapor molecules in a given volume of the atmosphere if the air were saturated*

The saturation vapor pressure depends only on temperature, and increases as temperature increases.

# Clausius-Clapeyron relationship



- Saturation vapor pressure depends only on temperature  
i.e.,  $e_s = e_s(T)$
- From this one can derive saturation mixing ratio, saturation specific humidity
- Also dew point temperature is the temperature at which  $e = e_s$   
i.e.,  $e = e_s(T_d)$



Blue curves are mixing ratio

Can be used to find/convert dew point temperature

Notice almost parallel to temperature, and a reminder that  $e_s = e_s(T)$

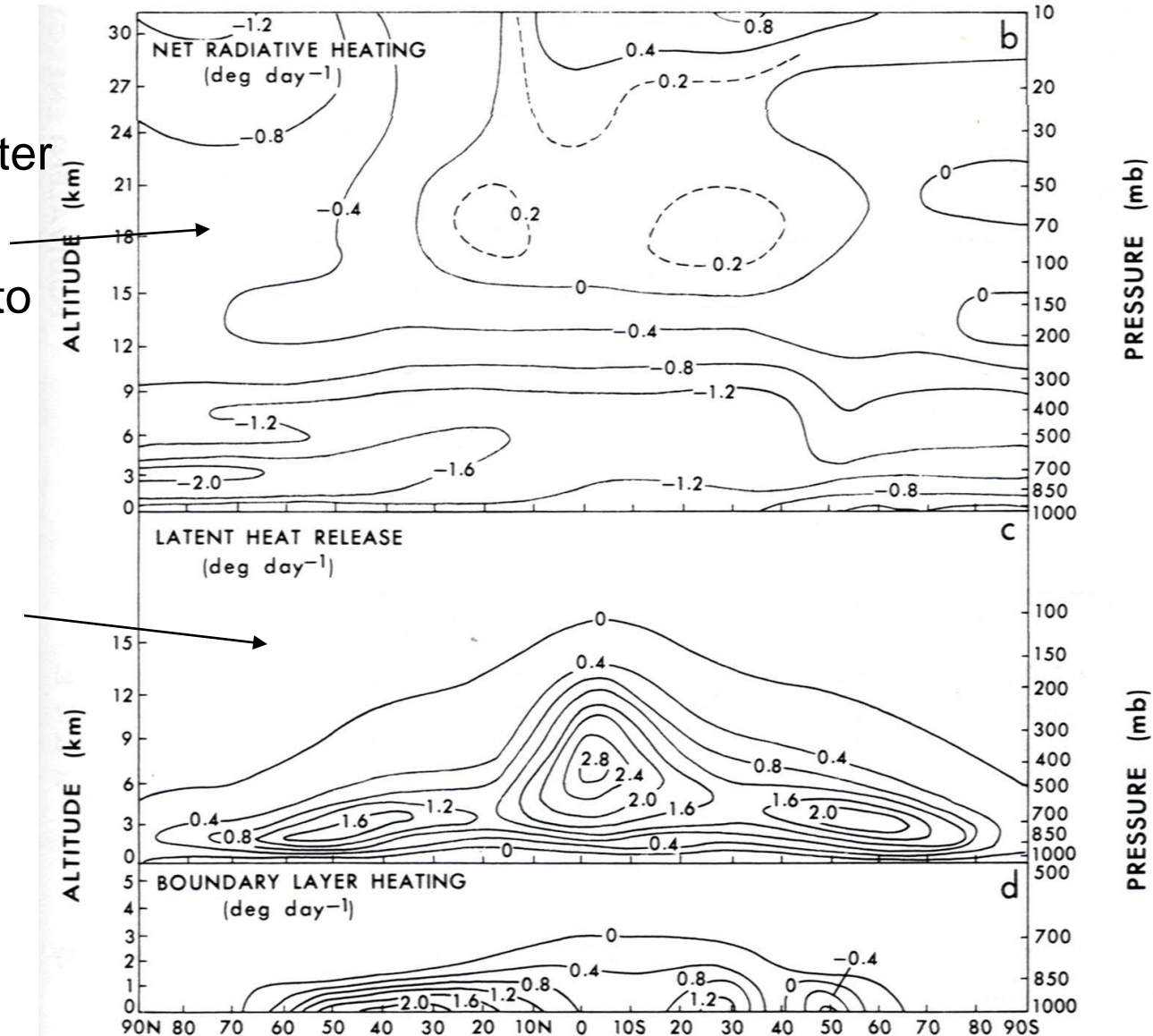
Or  $q_s = q_s(T,P)$

# Latent heating

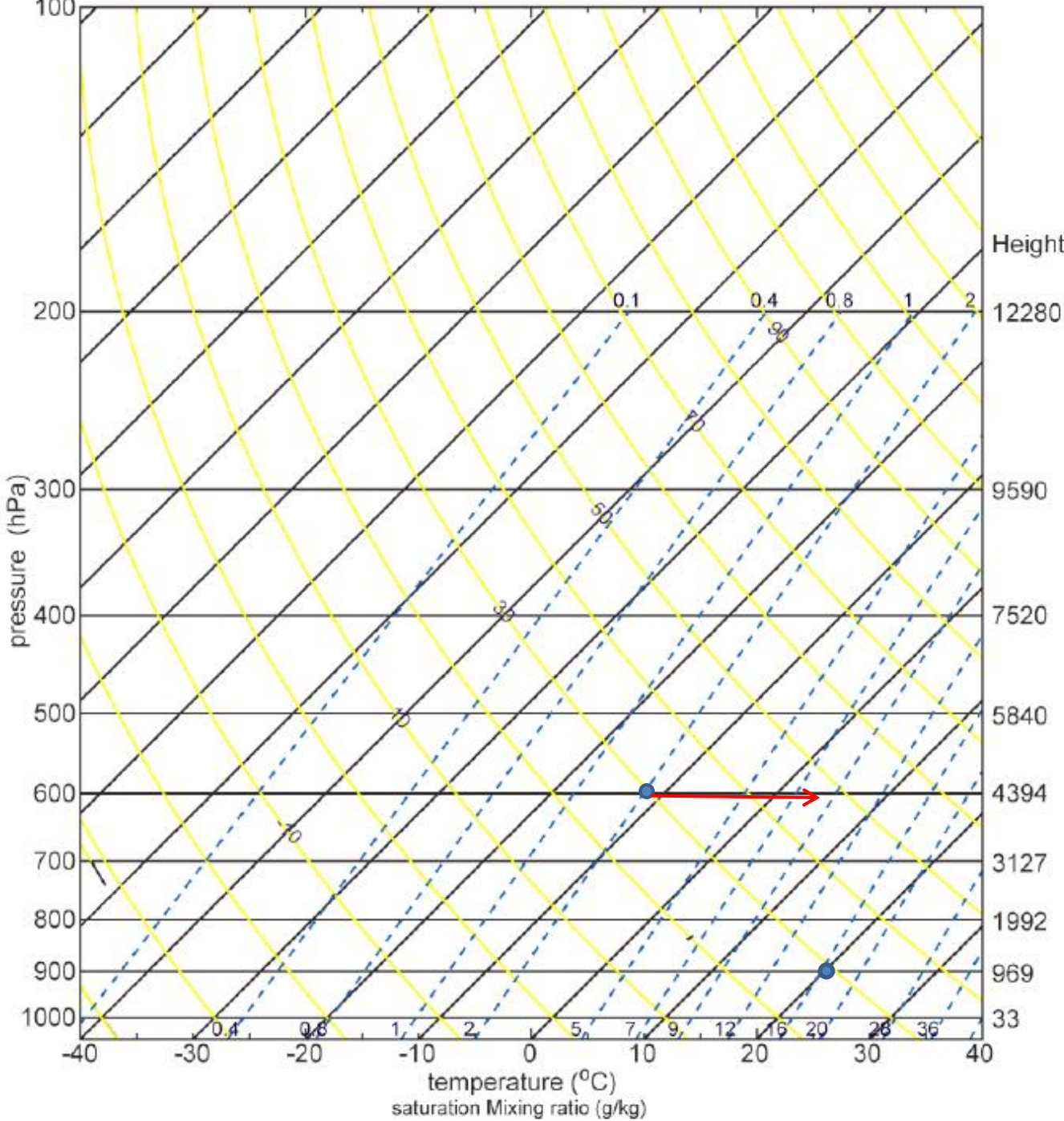
- Latent heating:  $J = Ldw$
- $L$  is the latent heat (of vaporization, or sublimation or freezing)
- E.g. how much energy is released when the mixing ratio in a 100 hPa layer decreases from 16g/kg to 2 g/kg because of (liquid) rain.  
 $L_v = 2.25 \times 10^6 \text{ J/kg}$        $m_{\text{air}} = \Delta p/g$
- $J = L_v \times \Delta w \times m_{\text{air}} = 2.25 \times 10^6 (16 - 2) \times 10^{-3} \times 10000 / 9.8 = 3.21 \times 10^7 \text{ J}$
- What if this is converted to heat?
- From the first law:  $c_p dT = L_v dw$
- So,  $dT = 2.25 \times 10^6 (16 - 2) \times 10^{-3} / 1005 = 31 \text{ K}$
- i.e., condensation heats the atmosphere... a lot

# Zonal mean heating terms

Absorption and emission from water molecules  
(also albedo due to clouds/ice/snow)



Due to condensation



Height (m)

12280

9590

7520

5840

4394

3127

1992

969

33

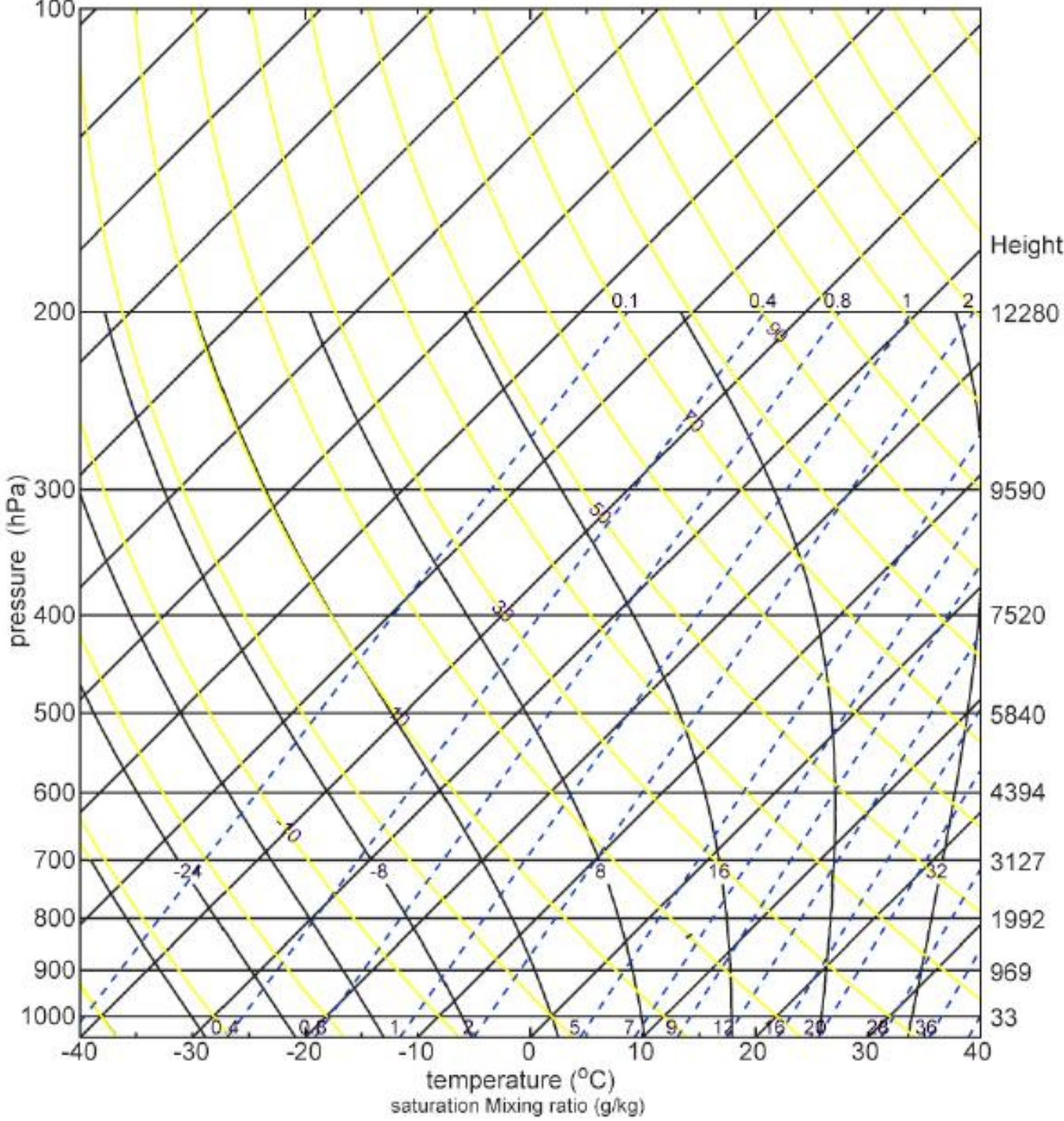
As parcel rises,  
condensation occurs.

Extra heat means ascent  
WILL NOT follow the dry  
adiabat.

Extra energy can do  
more work on the  
environment

# Moist adiabatic processes

- Consider a parcel lifted under saturated conditions
- Ascent means condensation will occur, and if condensation remains with the air parcel (as, say, cloud water) the processes is *reversible*
- *If condensation is formed in this way, and some time later fall as precipitation this is called pseudoadiabatic.*
- *From the first law...*



Height (m)

Most adiabats (black)

Curve upward faster than dry adiabats due to heating during ascent

Become parallel with dry adiabats at high altitude when water vapor concentration is low

## **Lifting condensation level**

- Height at which a parcel initially at the surface becomes saturated when lifted (dry) adiabatically
- Often sets the cloud base height

## **Level of free convection**

- The height at which the temperature of a rising air parcel ( $T_{parcel}$ ) first exceeds the temperature of the environment ( $T_{env}$ ).
- *Follow the moist adiabat above the LCL*
- *Often near base of convective cloud.*

## **Level of neutral buoyancy**

- Height at which a parcel rising (moist) adiabatically, reaches the same temperature as the environment.
- Often at top of convective storms near the tropopause, and forms the anvil as clouds detrain

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100

200

300

400

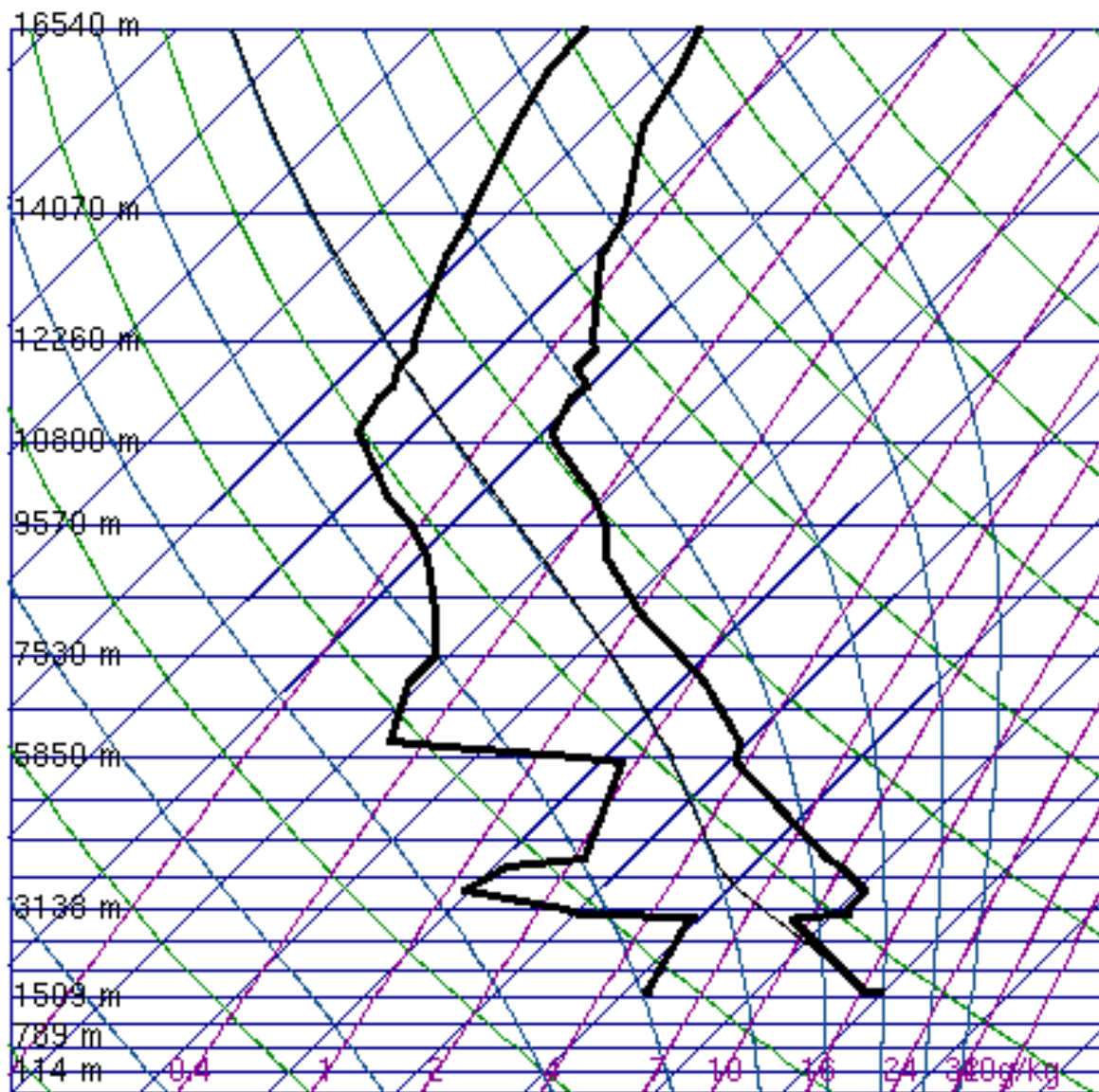
500

600

700

800

900

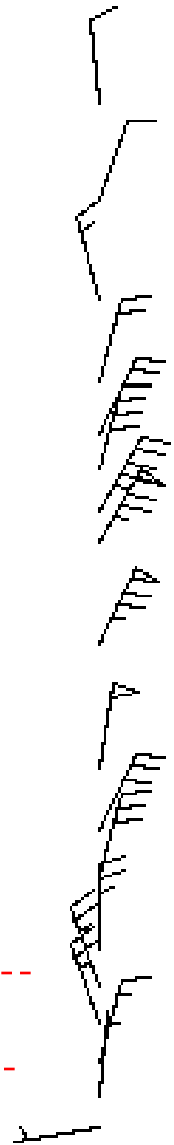
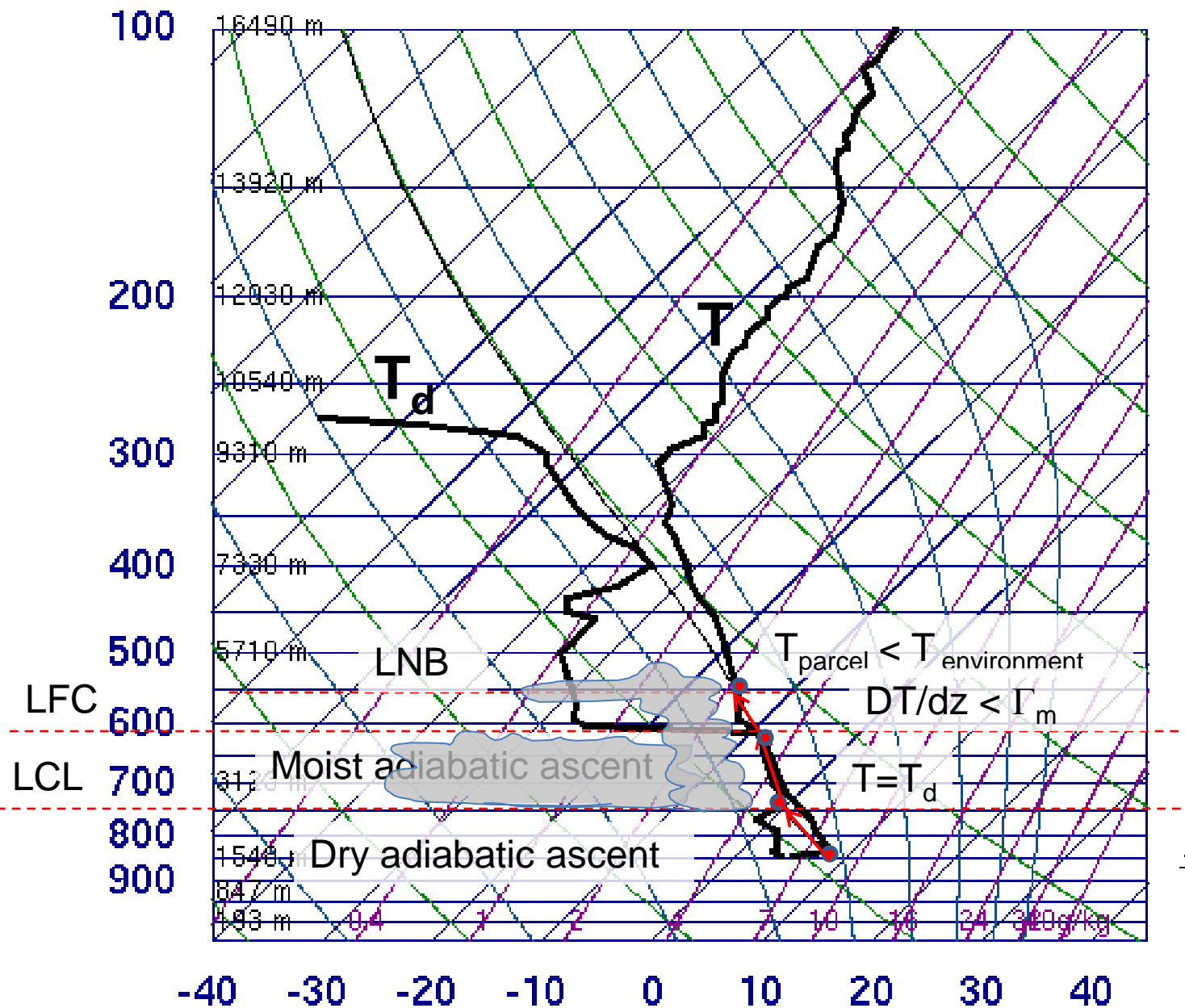


SLAT	39.75
SLON	-104.87
SELV	1625.
SHOW	-9999
LIFT	4.97
LFTV	4.80
SWET	-9999
KINX	-9999
CTOT	-9999
VTOT	-9999
TOTL	-9999
CAPE	0.00
CAPV	0.00
CINS	0.00
CINV	0.00
EQLV	-9999
EQTV	-9999
LFCT	-9999
LFCV	-9999
BRCH	0.00
BRCV	0.00
LCLT	271.1
LCLP	645.9
MLTH	307.2
MLMR	5.15
THCK	5736.
PWAT	11.40

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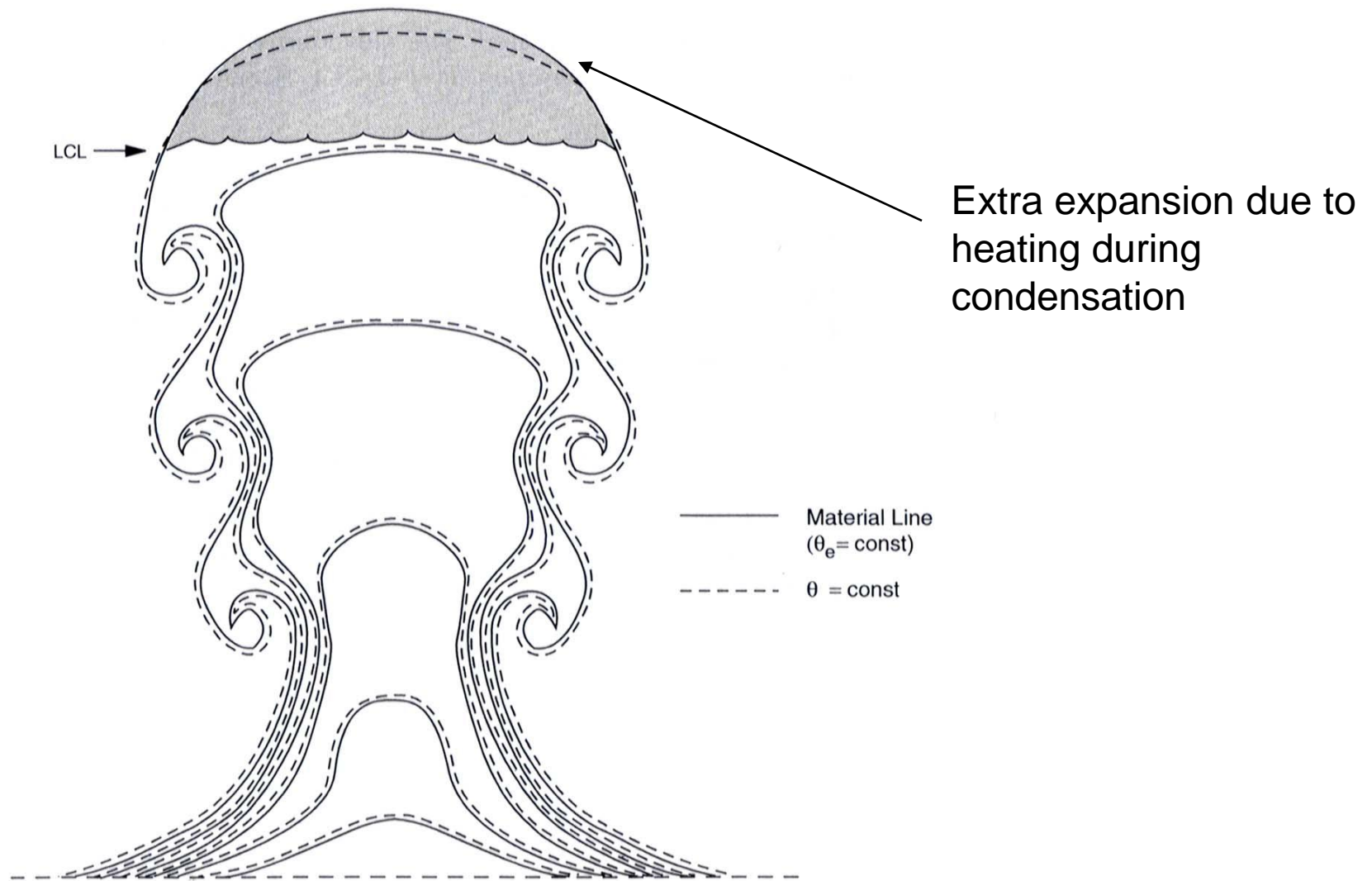
University of Wyoming

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SLAT	3
SLOE	-3
SELV	1
SHOW	-9
LIFT	1
LFTV	0
SWET	-9
KINX	-9
CTOT	-9
VTOT	-9
TOTL	-9
CAPE	1
CAPV	1
CINS	-9
CINV	-9
EQLV	5
EQTV	5
LFCT	6
LFCV	6
BRCH	0
BRCV	0
LCLT	2
LCLP	7
MLTH	2
MLMR	5
THCK	5
PWAT	1

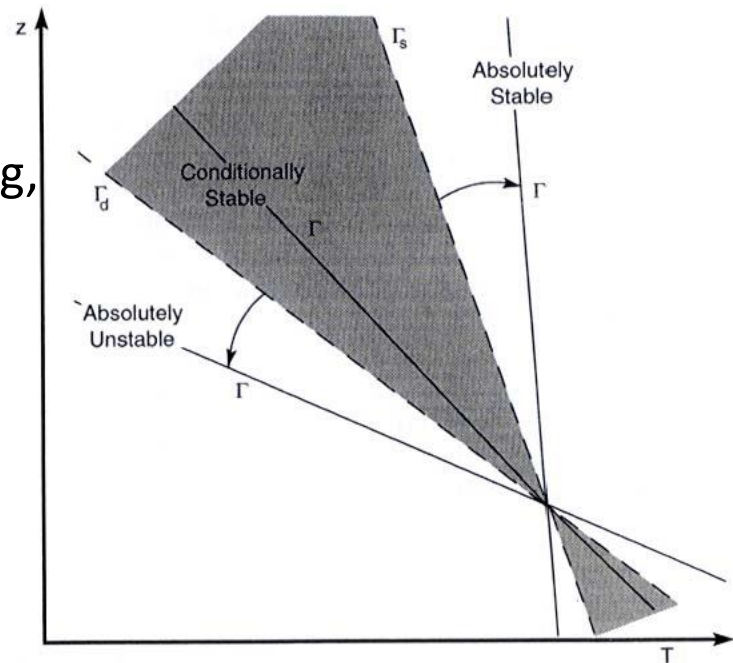
# Enhanced buoyancy with condensation

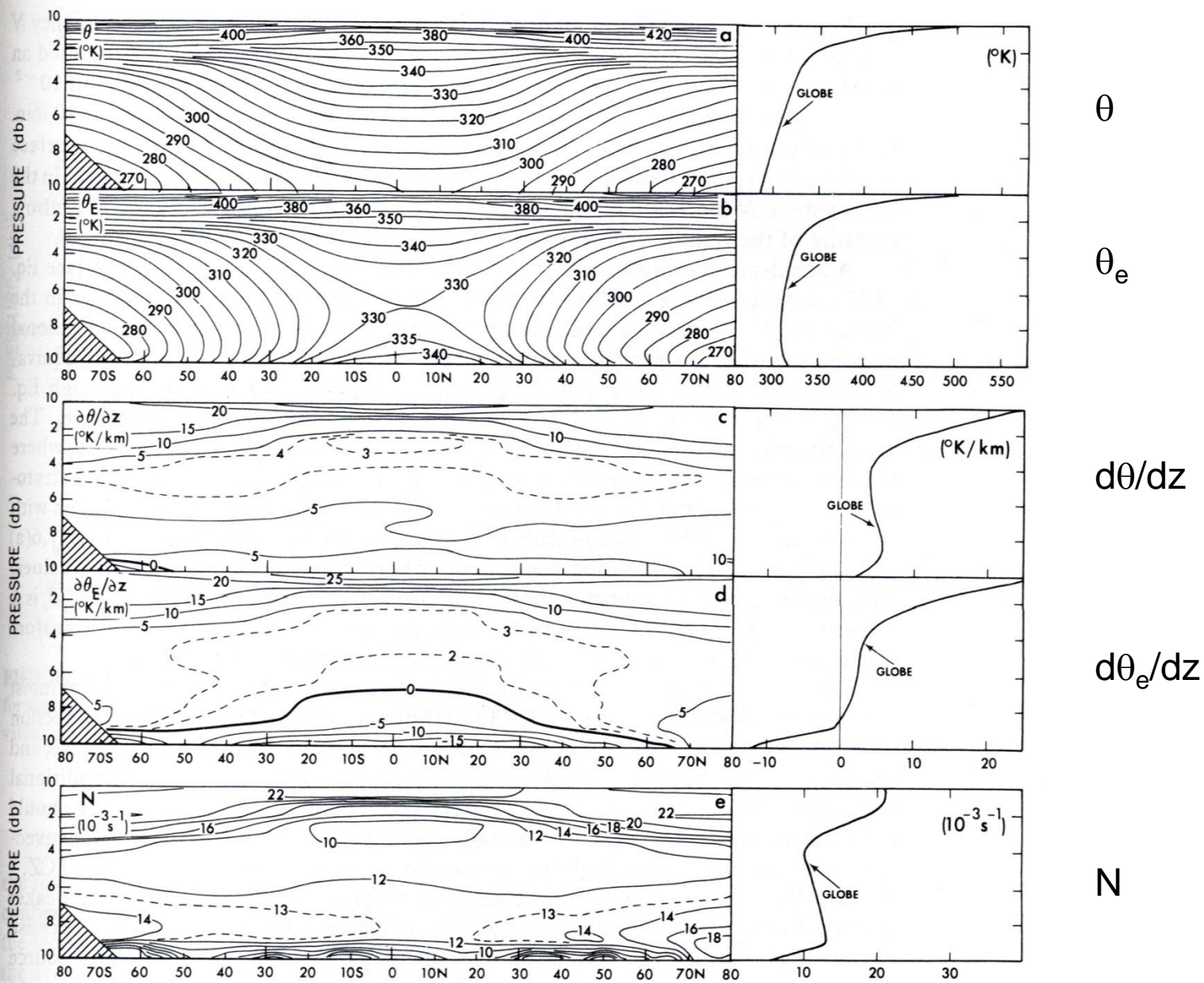


# Moisture and buoyancy

- As a moisture condenses it changes the parcel heat from latent to a sensible form which can effect volume and density.  
(i.e., we can replace  $\theta$  with  $\theta_e$ )
- As such, the lapse rate a parcel follows changes from dry to moist adiabatic.
- As the moist adiabatic lapse rate is a weaker constraint, the buoyancy may increase
- So we have additional conditions for determining the stability of the environmental profile:

- 1)  $\Gamma < \Gamma_s$  absolutely stable  
(positive restoring, condensation of not)
- 2)  $\Gamma > \Gamma_d$  absolutely unstable (negative restoring, condensation or not)
- 3)  $\Gamma_s < \Gamma < \Gamma_d$  conditionally stable (positive restoring while unsaturated, but negative restoring if condensation occurs)





**FIGURE 7.6.** Zonal-mean cross sections of the potential temperature (a) in K, equivalent potential temperature (b) in K, vertical gradient of potential temperature (c) in K/km, vertical gradient of equivalent potential temperature (d) in K/km, and Brunt-Väisälä frequency in  $10^{-3} \text{ rad s}^{-1}$  for annual-mean conditions. Vertical profiles of the global mean values are shown on the right.

# Convective instability

- Conditional stability refers to the parcel somewhere in the environment
- Define convective instability (or potential instability) that is unstable only if the parcel is sufficiently modified by displacement.
- Should a displacement allow condensation, the parcel can extract additional energy from the profile
- Energy is released - can do (expansion) work, or be converted to kinetic energy (this is the convective available potential energy, CAPE)
- Rather than considering the lifting condensation level (LCL), we also have a level of free convection (LFC) on a thermodynamic chart
- Very energetic system, e.g. where there is substantial amounts of CAPE or as the profile is very unstable, the buoyancy is greatly enhanced and deep convective plumes. E.g. tropical storms, Supercells of Oklahoma, afternoon storms over Boulder
- However, this requires some kind of trigger to allow the release of CAPE