Coriolis

Newton’s laws

- *First Law*: no forces = no acceleration
  (parcels that balance gravity and centrifugal forces must be on geopotential surfaces to be at rest)

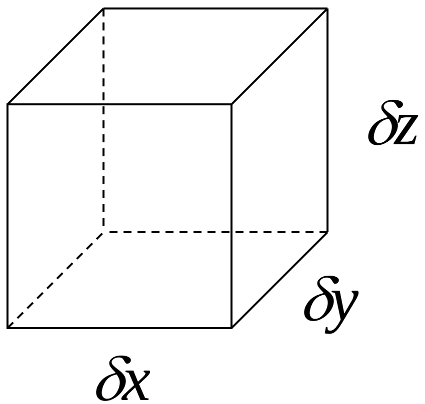
- *Second Law*: forces change momentum

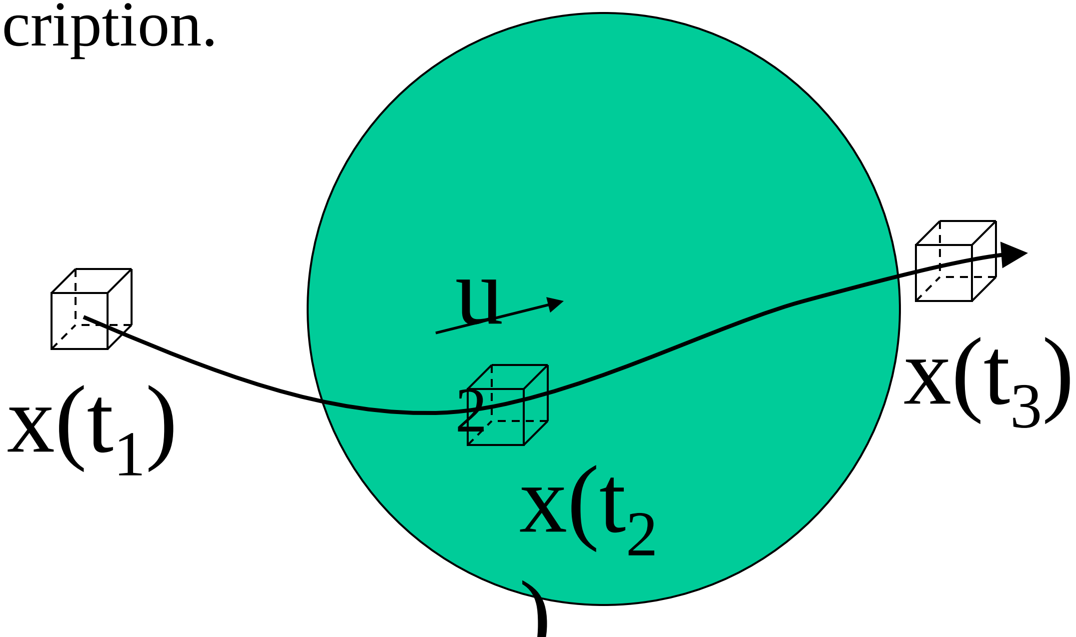
- *Third Law*: action = reaction
  (e.g., centrifugal = *minus* centripetal)

- These apply to all air parcels

Air parcels (1)

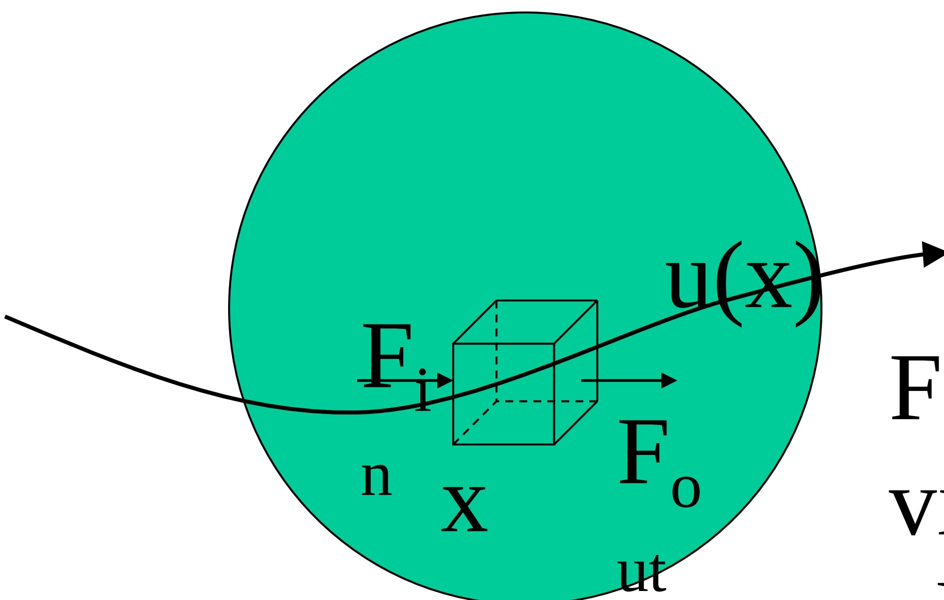
- Imagine a parcel consists of a fixed number of fluid particles (say molecules) – the material (mass) is conserved
- The parcel moves along with the flow (imagine a balloon)
- External influences (such as forces) can change the parcel’s behavior (or properties, such as temperature)
- This change occurs as the parcel moves along it’s flow trajectory
- The volume conserves the material that makes it up.
- This the *Lagrangian* description.





Air parcels (2)

- Alternatively, we can imagine the parcel to be fixed in space
- Fluid flows into the parcel on one side and out of the parcel on the other side, changing the properties of the parcel
- Again local influences can change parcel properties
- This is the *Eularian* viewpoint



From both viewpoints the physics is the same

Derivatives – total and partial

- Rate of change of some property, *q*, for a parcel of air (or other body), dq/dt
- Parcel is at some (3d) position (x, y, z)
  Position also changes in time (4^{th} dimension is t)

$$x = x(t),\ y = y(t),\ z = z(t)$$

- Similarly,

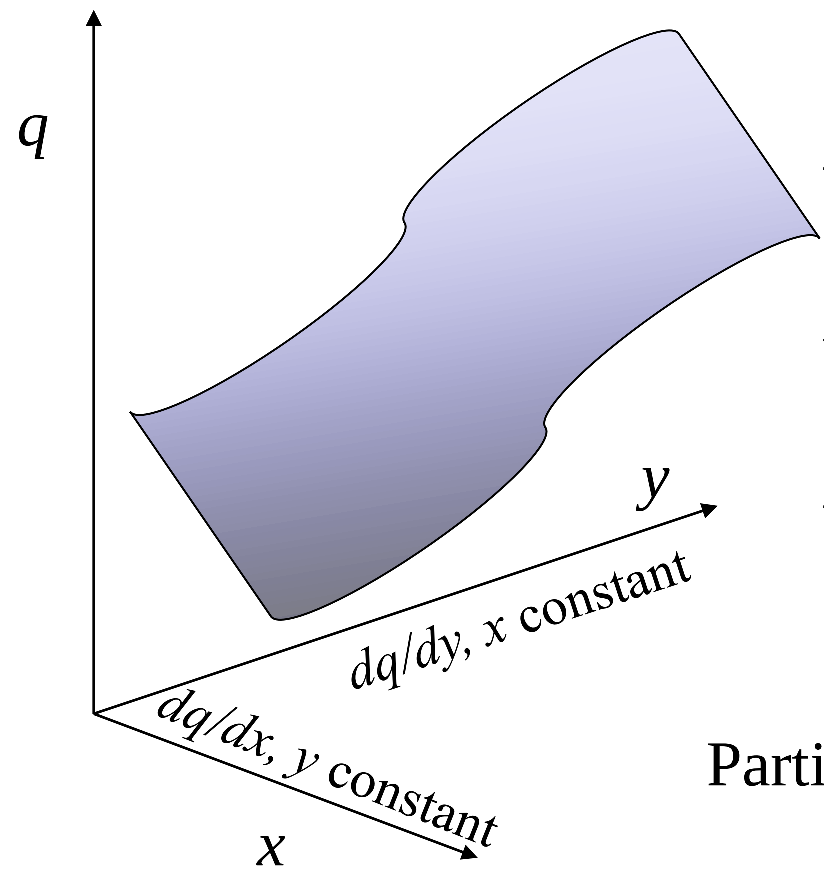
$$u = dx/dt,\ v = dy/dt,\ w = dz/dt$$

Thus for the total derivate of *q*, must account for the (partial) contributions from changes in all dimensions (x, y, z, t).

(We have already seen an example of this when transforming vertical coordinates.)

Total derivatives

- Geometrically we can write the total derivative in terms of the sum of the partial derivatives



$$\delta q = \delta t\left(\frac{\partial q}{\partial t}\right) + \delta x\left(\frac{\partial q}{\partial x}\right) + \delta y\left(\frac{\partial q}{\partial y}\right) + \delta z\left(\frac{\partial q}{\partial z}\right)\ldots$$

$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + \frac{dx}{dt}\left(\frac{\partial q}{\partial x}\right) + \frac{dy}{dt}\left(\frac{\partial q}{\partial y}\right) + \frac{dz}{dt}\left(\frac{\partial q}{\partial z}\right)$$

$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + u\frac{\partial q}{\partial x} + v\frac{\partial q}{\partial y} + w\frac{\partial q}{\partial z}$$

$$\frac{\partial q}{\partial t} = \frac{dq}{dt} - \mathbf{V} \bullet \nabla q$$

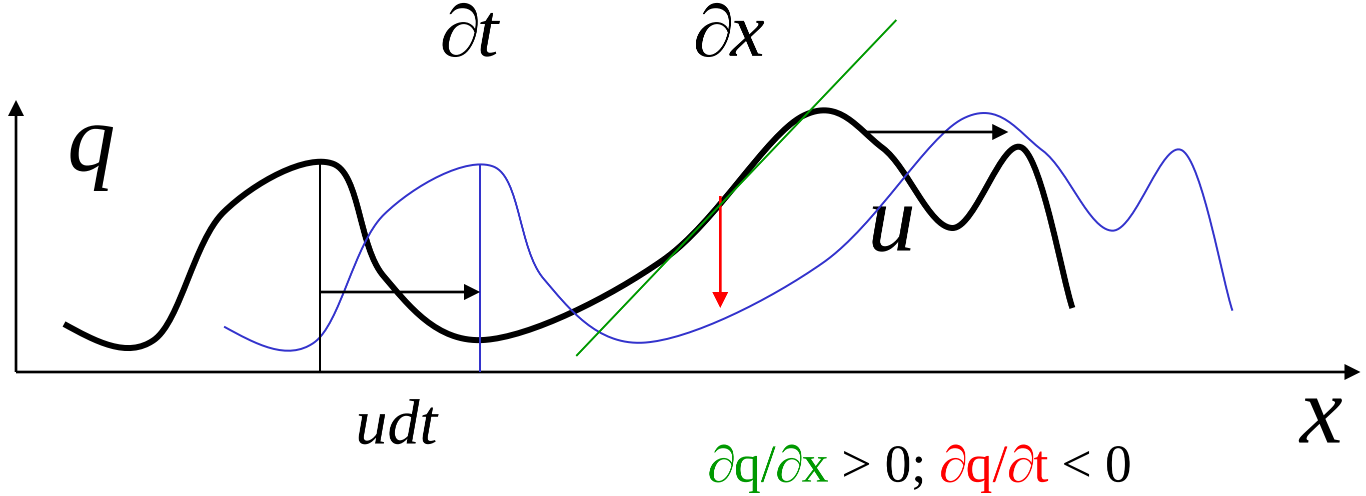
Advection

Partial derivatives fixed in space (x,y,z fixed)

Total derivatives follow the motion

Advection – physical interpretation

$$\frac{\partial q}{\partial t} = -u \frac{\partial q}{\partial x}$$



Advection describes the transport of a quantity into parcel/region of air fixed in place.

i.e., a translation of the pattern (surface),

no modification following the motion

Total derivatives for Lagrangian view (following the motion)

Partial derivatives give Eularian view (fixed in space)

Example

- Greeley is 10K warmer than Boulder
- The wind is blowing from the northeast
  (as when a low pressure system is near-by)
- Boulder becomes warmer due to temperature advection
- Quantitatively, warming estimated from temperature gradient, and wind speed.

- This specific example, this is also related to a change in moisture (due to advection). Upon hitting the mountains of the Front Range, the air ascends cools, and snows.

  * Watch for this in winter to predict good skiing!

Exercise 1.2

Trace gases - water and ozone

For all air parcels, the only change in atmospheric water is due to the difference between evaporation "source" and condensation (precipitation) "sink"

$$\frac{dq}{dt} = \text{source} - \text{sink}$$

$$\frac{\partial q}{dt} = -\left(u\frac{\partial q}{dx} + v\frac{\partial q}{dy} \right) + (\text{source} - \text{sink})$$

Same equation for ozone! Now the source and sink is dominated by photolysis, dissociation, chemical loss.

In both cases, on short time scales, the advection term usually dominates

General form of transport equation

- Before, we showed a general equation

  $\partial q / \partial t = -\mathbf{V} \cdot \nabla q$ + (source-sink)
- Can we use this for momentum?
  What are the sources and sinks for momentum?
- What about temperature?

We are motivated to come up with a quantity related to temperature and momentum that has no source and sinks, so we can use the transport equation for dynamics (we’ll see in future lectures this is *potential vorticity*)

Recall, forces

Real ones:

- Gravity, pressure gradient, (viscous)

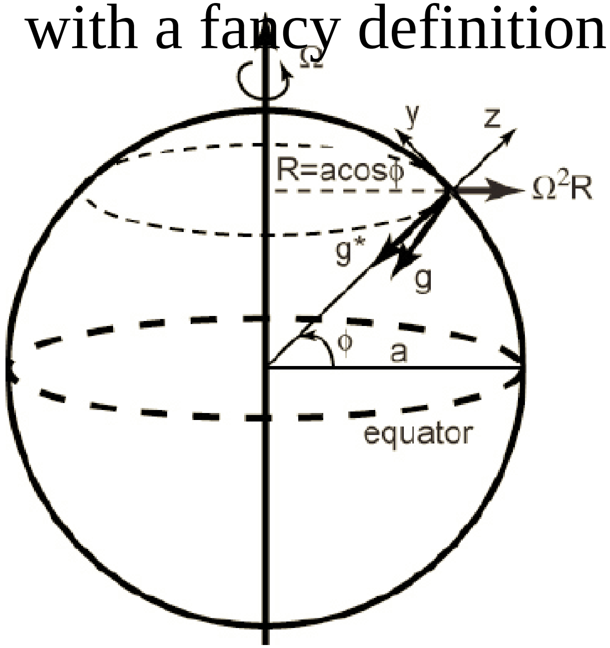
Apparent ones

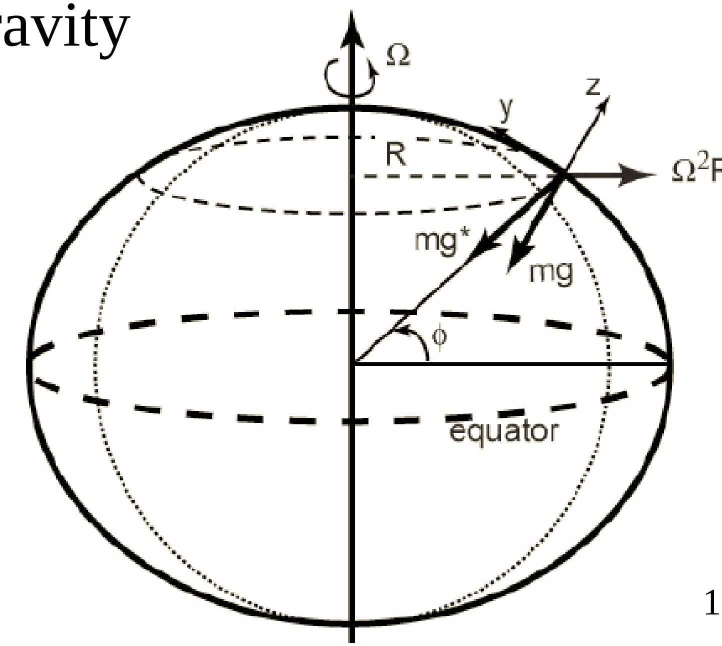
- Centrifugal, Coriolis

Body forces

- Consider a ball sitting on the surface of a spherical earth.
- Gravity – always down, toward center of mass
- Centrifugal – always outward from rotation axis
  Define absolute, planetary and relative: $\Omega_{total} = \Omega_{planet} + \omega_{air}$
  (strictly, we're only considering the planetary part here)

- It is convenient to combine these…
  with a fancy definition of gravity





Define gravity

- Direction of acceleration due to "gravity" is that directed perpendicular to a "geopotential" surface

- Specifically, **g** = **g*** + Ω**R**
  In vector form, or g is defined in the *k*, *(i.e, z)* direction (one may say k is defined by the direction of **g**!)
- Consider the component of the centrifugal force which projects onto the local vertical

- Thus define geopotential via gravity as
  $$g = -\nabla\Phi$$

- So , Φ = Φ(z) and one may write $$\frac{\partial \Phi}{\partial z} = g$$

Consider momentum again:

•

- **1. Using conservation of angular momentum (*M*), *what is the zonal (westerly) velocity of a parcel initially at rest at the equator when displaced to 30 N?***
- *Hint:* $M = (\Omega + u/R)R^2$,

u relative (westerly) velocity

R the distance to the rotation axis
(relative angular velocity is u/R)

Earth is spherical, so $R = a \cos \phi$

a radius of the Earth = 6,391,000 m

Ω Rotation rate 2π radians per day
i./e., $2\pi/(24 \times 60 \times 60) = 7.272 \times 10^{-5}\ s^{-1}$

Influence of rotation on moving parcels

Atmosphere (air parcels) conserve angular momentum

- Consider angular momentum per unit mass (A) (which is just the angular velocity)

$$A = (\Omega + u/R)R^2$$

Ω planetary rotation rate
(2 π radians per day for Earth)

u relative (westerly) velocity

R the distance to the rotation axis
(relative angular velocity is u/R)

Earth is spherical, so $R = a \cos \phi$

Since Ω is constant, if we change latitude ϕ, we must also change the velocity of the parcel.
This is what we wish to quantify.

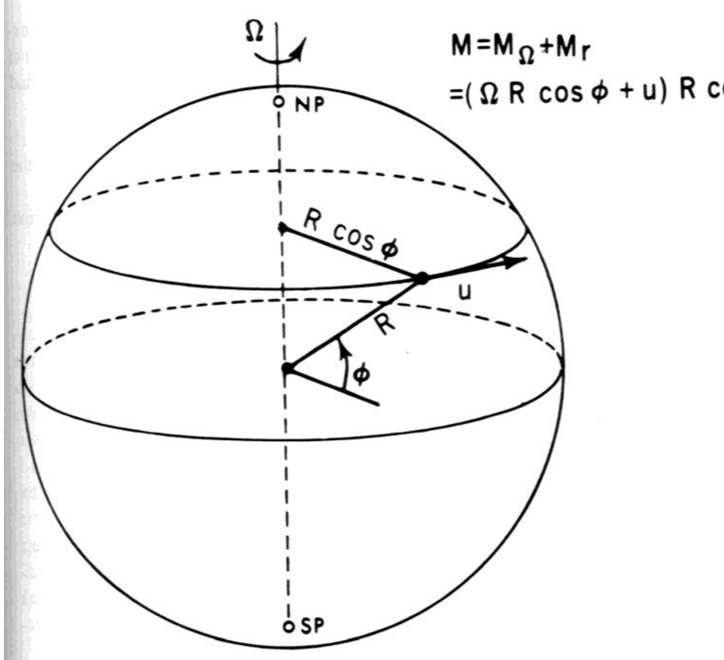


FIGURE 11.1. Schematic diagram of the angular momentum component around the ea of rotation. NP = North Pole; SP = South Pole.

Example

- Using conservation of angular momentum, what is the zonal (westerly) velocity of a parcel initially at rest at the equator when displaced to 30 N?

- Hint: $A = (\Omega + u/R)R^2$

$a = 6{,}391{,}000$ m, $\Omega = 2$ pi radians per day

Large scale structure of the atmosphere

Meridional circulation

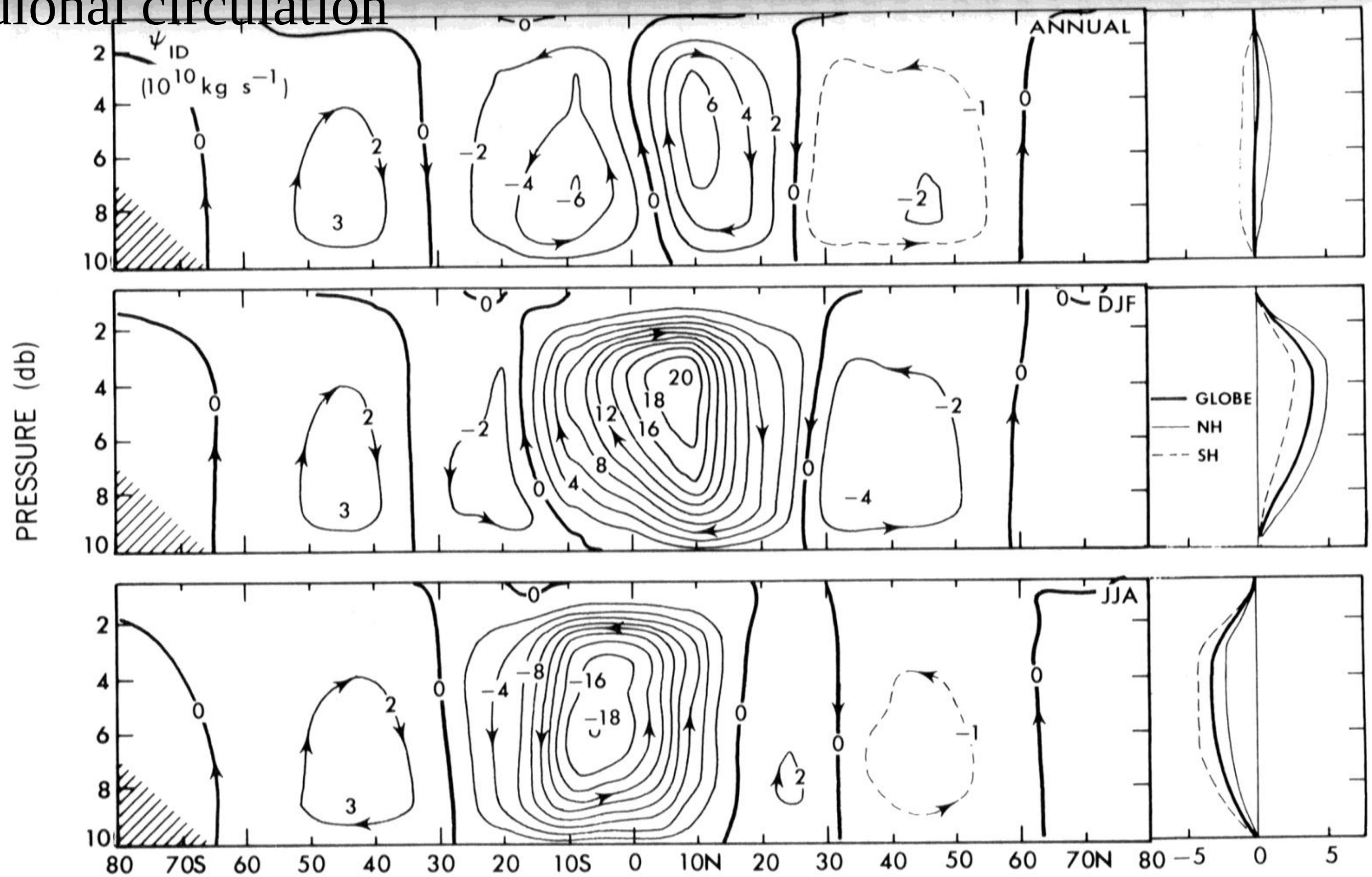
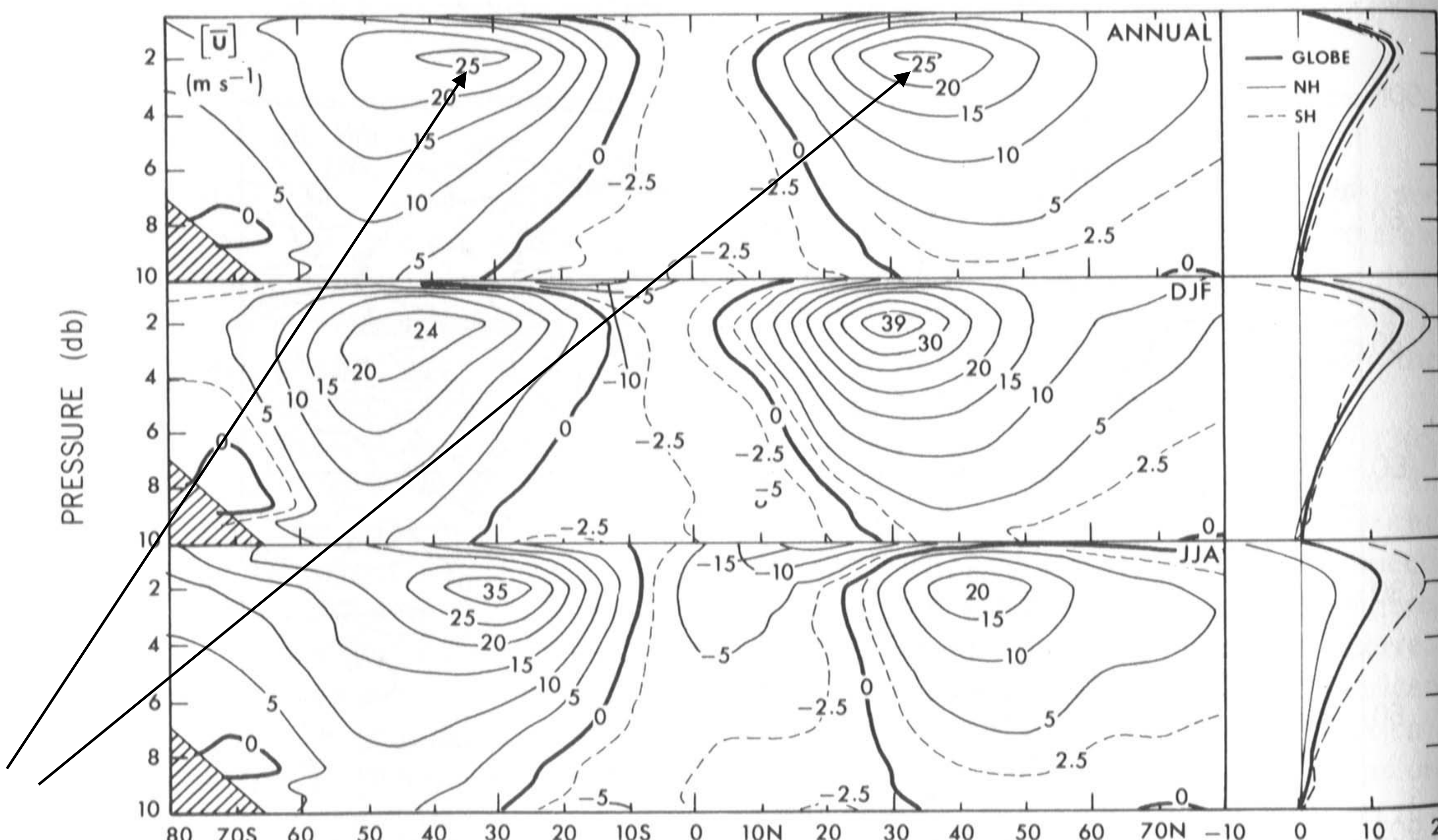


FIGURE 7.19. Zonal-mean cross sections of the mass stream function in 10^{10} kg s^{-1} for annual, DJF, and JJA mean conditions. Vertical profiles of the hemispheric and global mean values are shown on the right.

Peixoto and Oort, *Physics of Climate*, page 159

Large scale structure of the atmosphere

Westerly wind, *u*



?

FIGURE 7.15. Zonal-mean cross sections of the zonal wind component in m s^{-1} for annual, DJF, and JJA mean conditions. Vertical profiles of the hemispheric and global mean values are shown on the right.

What did we do wrong?

Peixoto and Oort, *Physics of Climate*, page 154

Coriolis effect

Deduce velocity change by equating angular momentum at two locations R and $R+\delta R$

$$\left(\Omega+\frac{u}{R}\right)R^2=\left(\Omega+\frac{u+\delta u}{R+\delta R}\right)(R+\delta R)^2$$

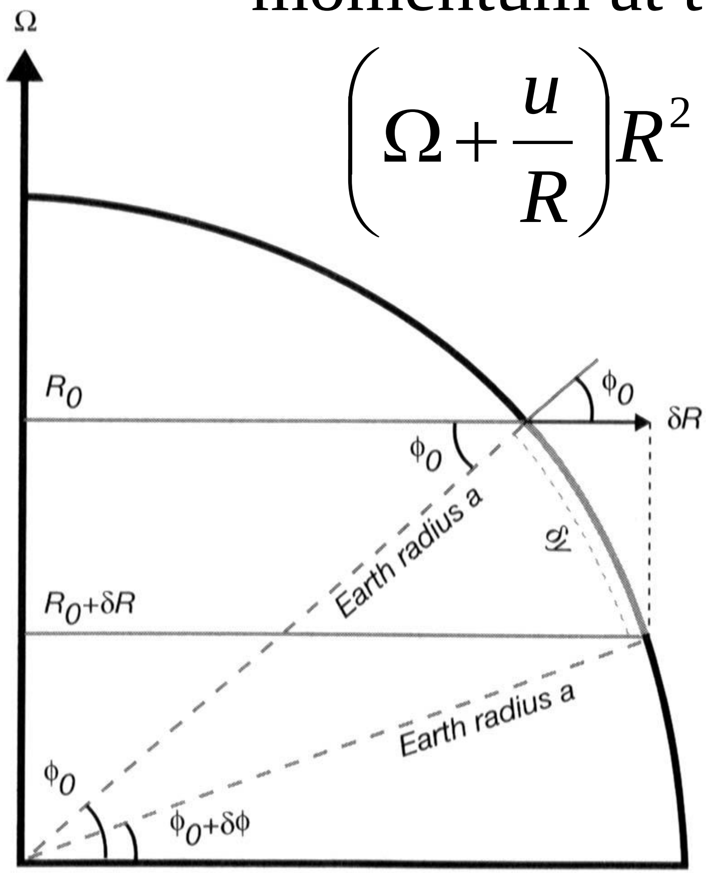


Fig. 1.8 Relationship of δR and $\delta y = a\delta\phi$ for an equatorward displacement.

δu is the deflection of the parcel

Because earth is spherical,

δR=-sin ϕ δy ***horizontal***

δR= *cos* ϕ δz ***vertical***

Coriolis force

After some algebra,

$$\frac{du}{dt} = 2\Omega v \sin\phi + \frac{uv}{a}\tan\phi$$ horizontal displacement

$$\frac{du}{dt} = -2\Omega w \cos\phi - \frac{uw}{a}$$ vertical displacement

Coriolis force   curvature terms

Similarly for southerly wind, *v*, and vertical wind, *w*

Coriolis parameter

- For most atmospheric motions the curvature terms are much smaller than the Coriolis term, so we can simplify. For ***horizontal*** motions,

$$\frac{du}{dt} = 2\Omega v \sin\phi = fv$$

$$\frac{dv}{dt} = -2\Omega u \sin\phi = -fu$$

Where f is the *Coriolis parameter*

$f = 2\,\Omega \sin\phi$ ($\approx 10^{-4}\,s^{-1}$ in mid-latitudes)

f is positive in NH and negative in SH

Coriolis force

1. Coriolis force is related to rotation, and thus directed relative to the rotation axis
2. We are often interested in the vertical component, since this effects horizontal motion
3. Coriolis forces cause flow to deflect to right of the motion in the NH, and to the left of the motion in the SH

$$\frac{F_{Co}}{m} = -2\Omega \times \mathbf{V}$$

Annual mean surface flow

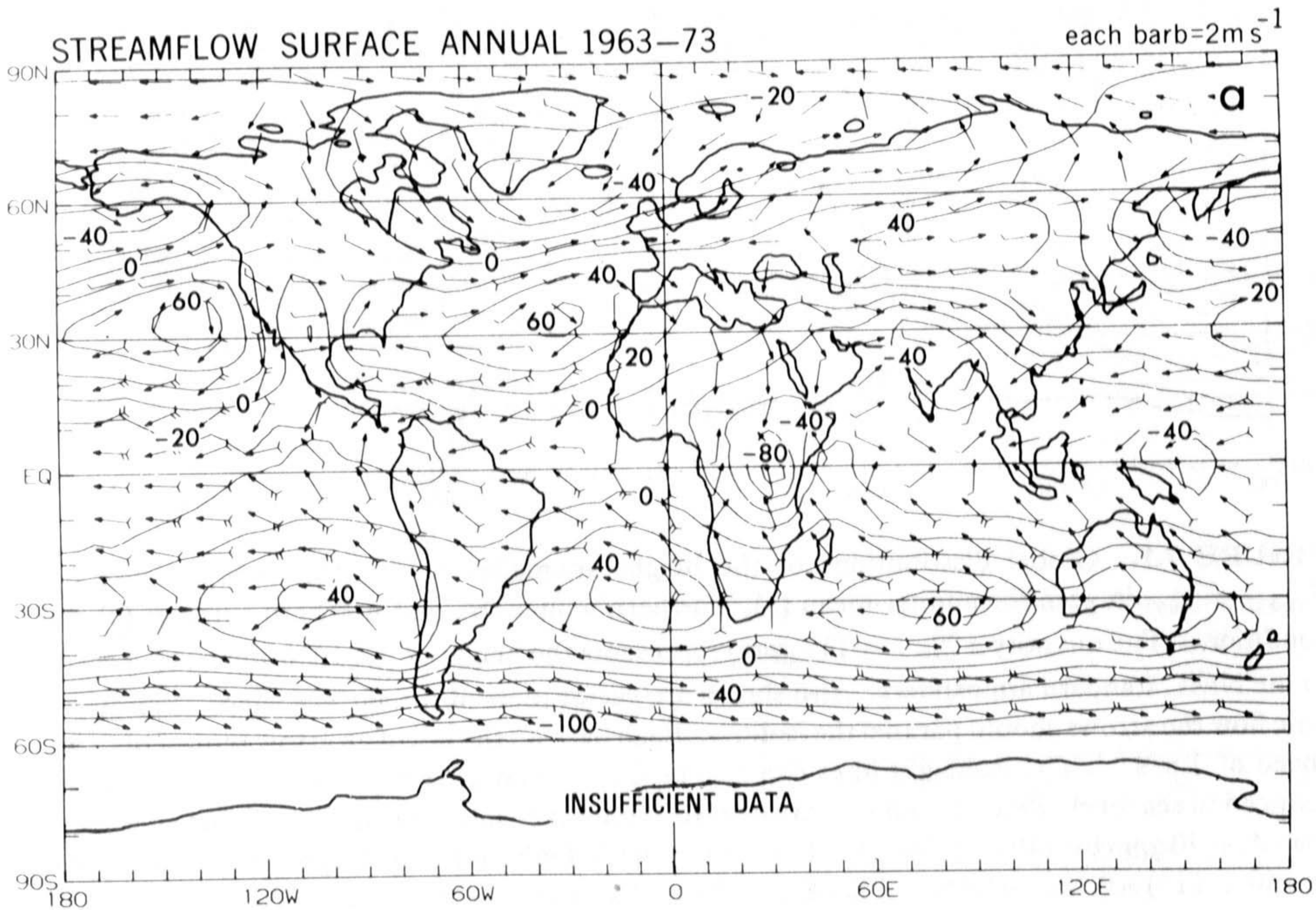


FIGURE 7.1a

Peixoto and Oort, *Physics of Climate*

Summary of forces

- Gravity
- Pressure gradient force
- Friction and viscosity
  (between layers of air, or at surfaces)

Also, electrostatic forces and magnetic forces
(These both act on ionized molecules, of which there are very few below ionosphere, so we can ignore them)

- Also have Coriolis force due to rotation and curvature

Our growing big nasty equation

- The momentum equation
  (conservation of momentum)

- Acceleration = pressure gradient + Coriolis
  (gravity in the vertical…. And sometimes friction)

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_{rx}$$

$f = 2\,\Omega \sin \phi$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_{ry}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

**Notice, I've neglected part of the Coriolis force…*

Exercise for Tuesday

- Holton 1.1
- Holton 1.8
- Holton 1.13