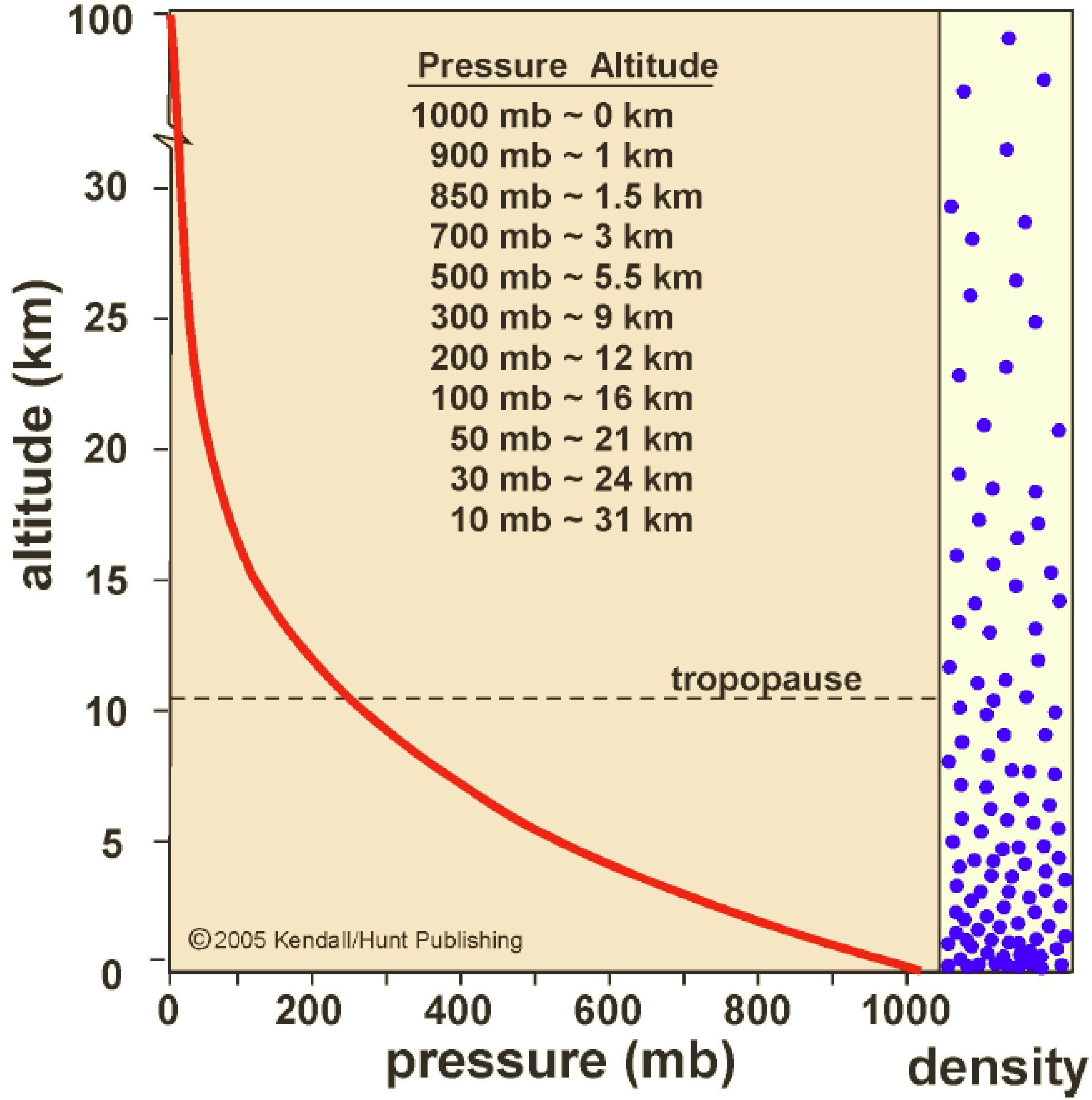
Forces and rotation

Composition of the atmosphere

- Air is a mixture of various gases
- Particularly, nitrogen, oxygen, water vapor, carbon dioxide
- We can assume air behaves like an ***ideal gas***

Relationship between temperature, pressure and density



Exosphere

Ionosphere

Thermosphere

Mesosphere

Stratosphere

Troposphere

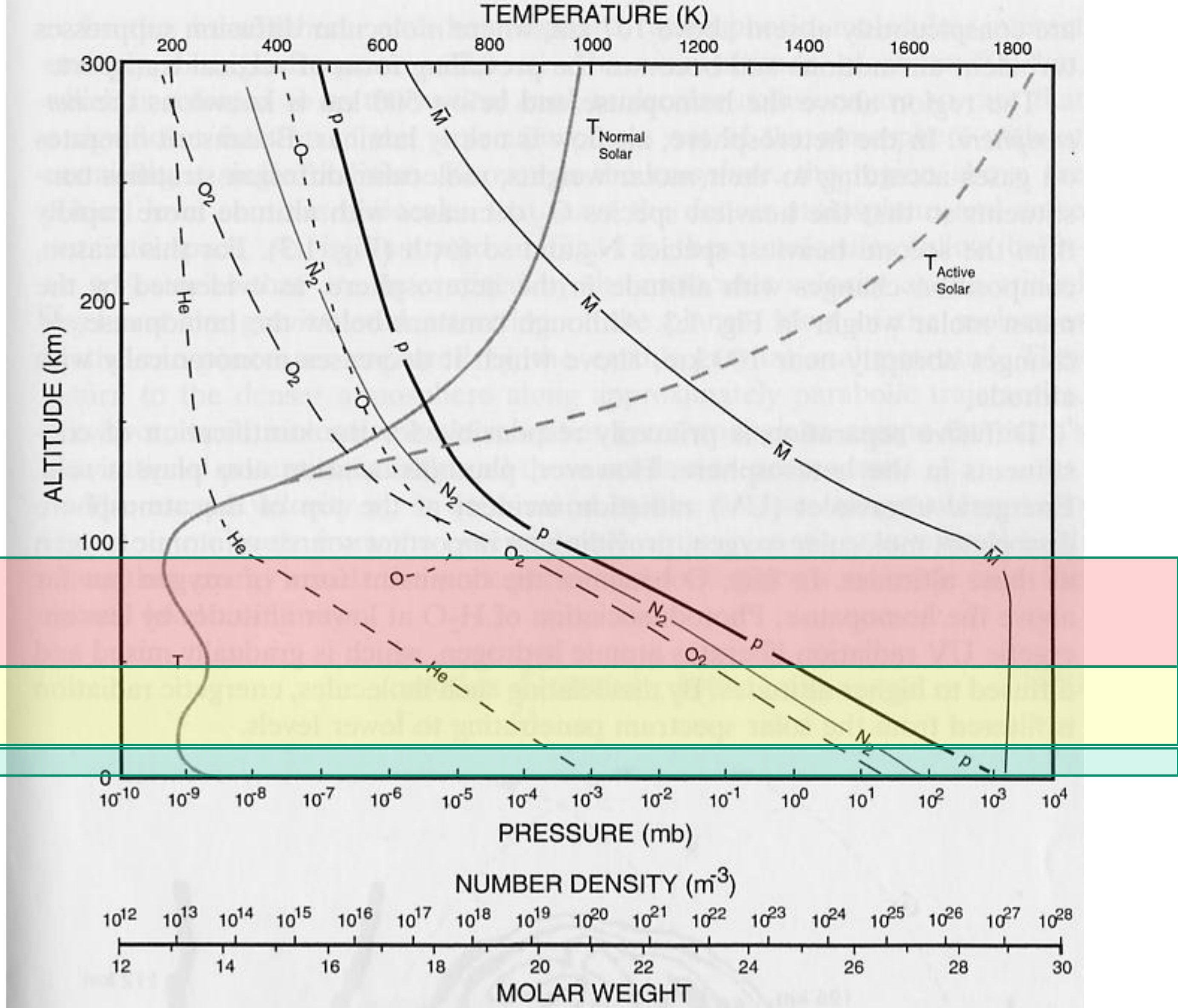


Figure 1.3 Global–mean pressure (bold), temperature (shaded), mean molar weight (solid), and number densities of atmospheric constituents, as functions of altitude. *Source:* U.S. Standard Atmosphere (1976).

Salby, *Fundamentals of Atmospheric Physics*, page 11.

Molecules absorb radiative energy

Radiative balance is a essential part of the climate system
Radiative imbalance makes the atmosphere move

Radiation comes in from the sun
(more in the tropics, less at high latitudes…
and only daytime)
Radiation goes out everywhere
("all things emit", land, clouds, atmosphere, trees, people….)

Radiation balance

- Radiation coming in from the sun, αS
- Radiation going out from the earth, L
- Global balance, Incoming = Outgoing, i.e., $\sum \alpha S = \sum L$

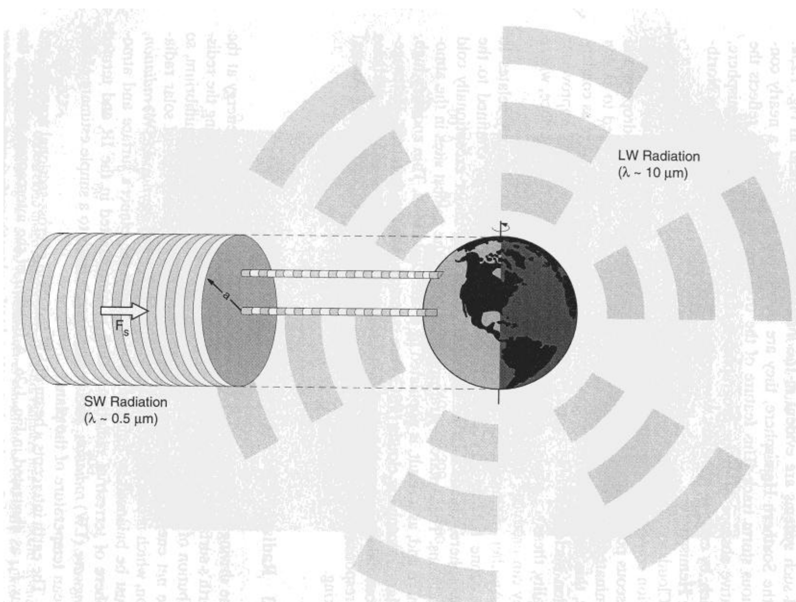


Figure 1.26 Schematic of SW radiation intercepted by the earth and LW radiation emitted by it.

At any point,

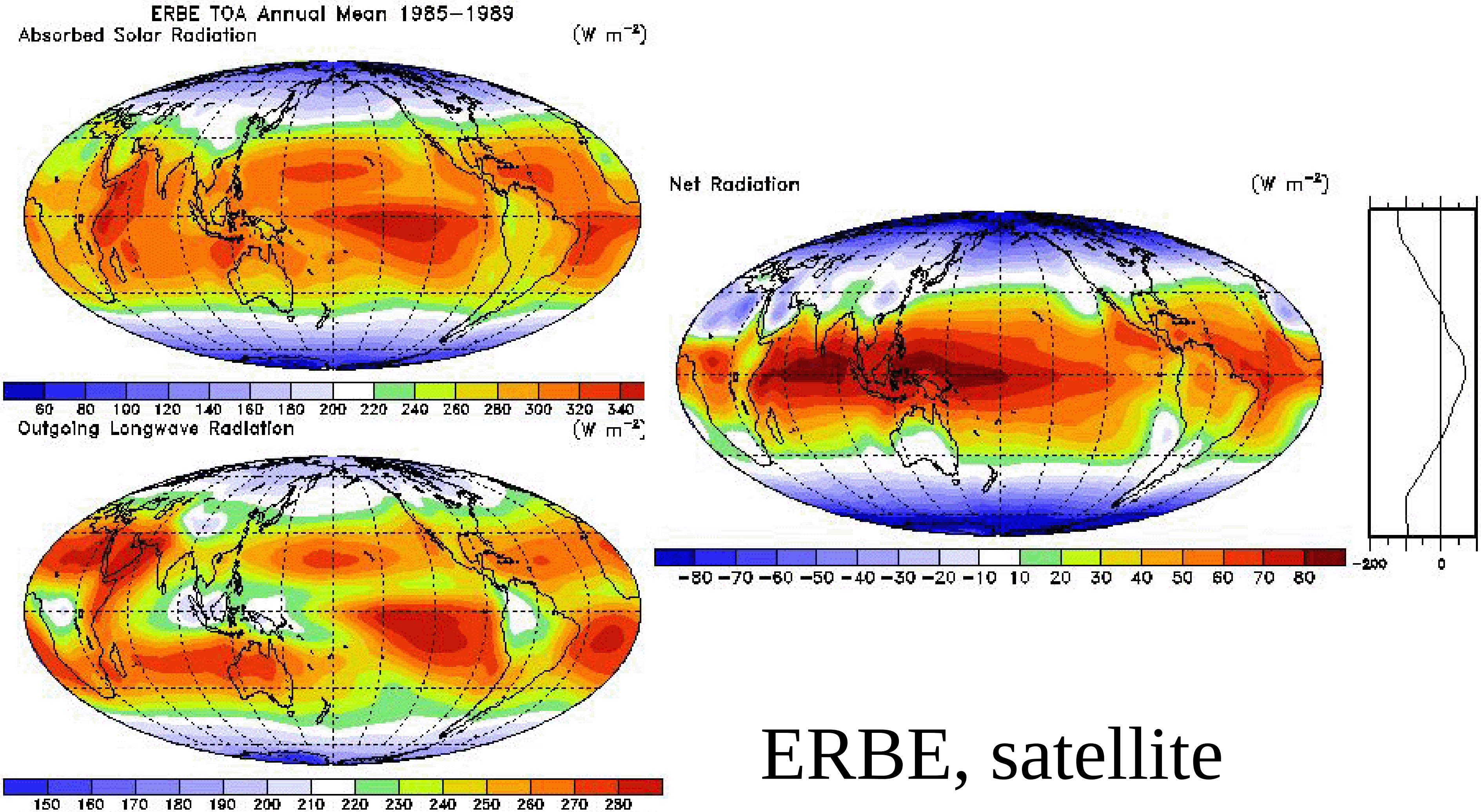
Incoming = $\alpha S \cos(\phi)$

Outgoing = σT^4

•However, for any point on the earth's surface, this need not be true. (consider, the amount of sunlight to the North Pole)

Salby, *Fundamentals of Atmospheric Physics*, page 42.

Radiation at the top of the atmosphere



ERBE, satellite measurements

Radiative profile

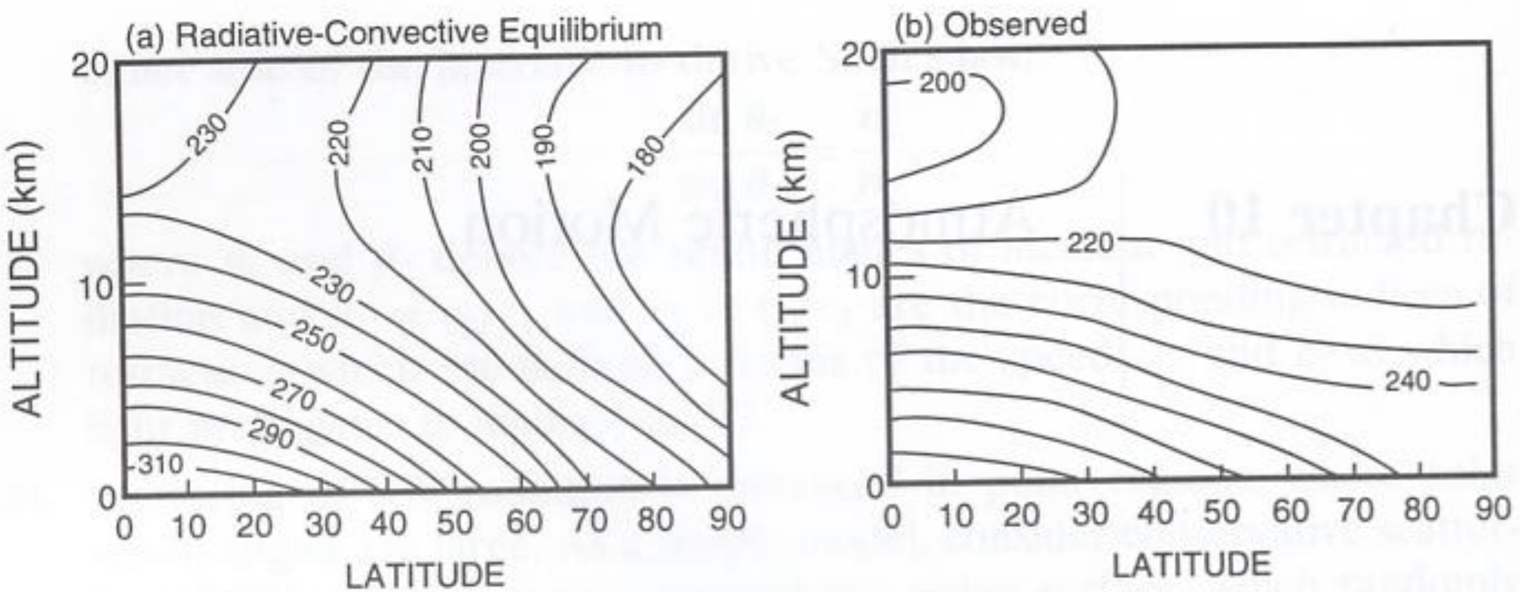


Figure 10.1 Zonal–mean temperature as a function of latitude and height (a) under radiative–convective equilibrium and (b) observed during northern winter. Without horizontal heat transfer, radiative–convective equilibrium establishes a meridional temperature gradient that is much stronger than observed. Sources: Liou (1990) and Fleming *et al.* (1988).

Salby, *Fundamentals of Atmospheric Physics*, page 322.

Large scale structure of the atmosphere

Temperature, *T*

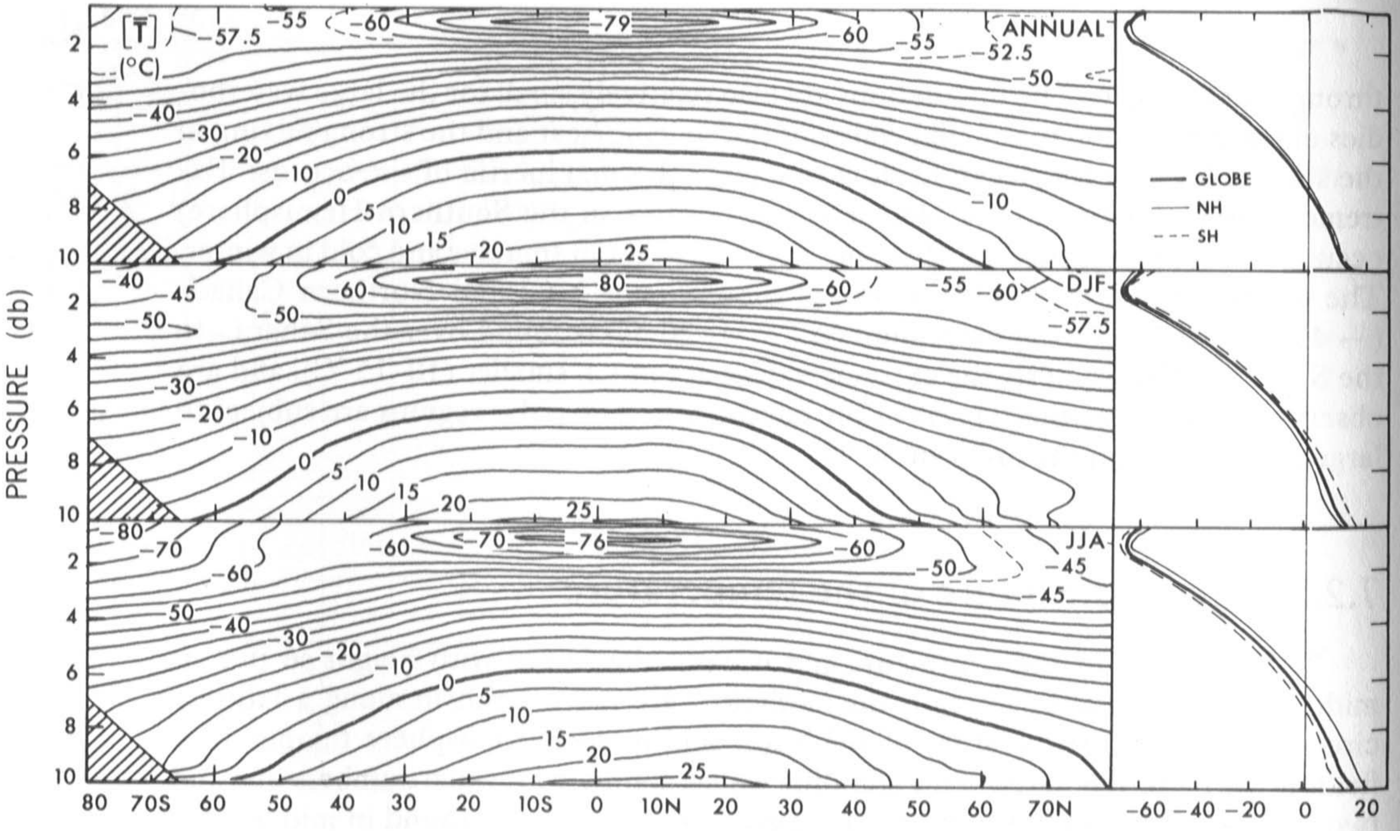


FIGURE 7.5. Zonal-mean cross sections of the temperature for annual-mean, DJF, and JJA conditions in °C. Vertical profiles of the hemispheric and global mean temperatures are shown on the right.

Radiative heating plus dynamics gives balanced mean state

Peixoto and Oort, *Physics of Climate*, page 140

Large scale structure of the atmosphere

Meridional circulation

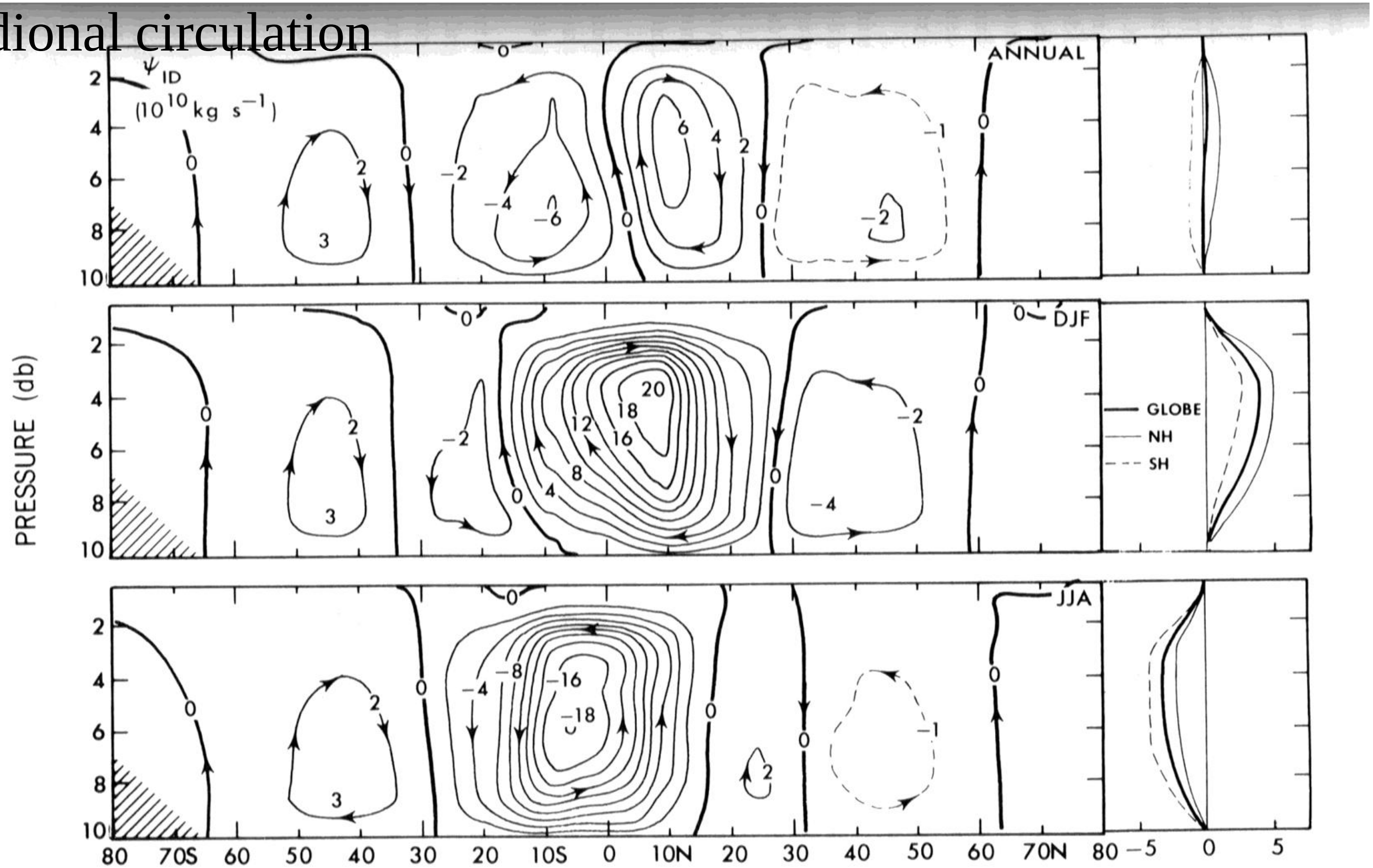


FIGURE 7.19. Zonal-mean cross sections of the mass stream function in 10^{10} kg s^{-1} for annual, DJF, and JJA mean conditions. Vertical profiles of the hemispheric and global mean values are shown on the right.

Peixoto and Oort, *Physics of Climate*, page 159

Large scale structure of the atmosphere

Westerly wind, *u*

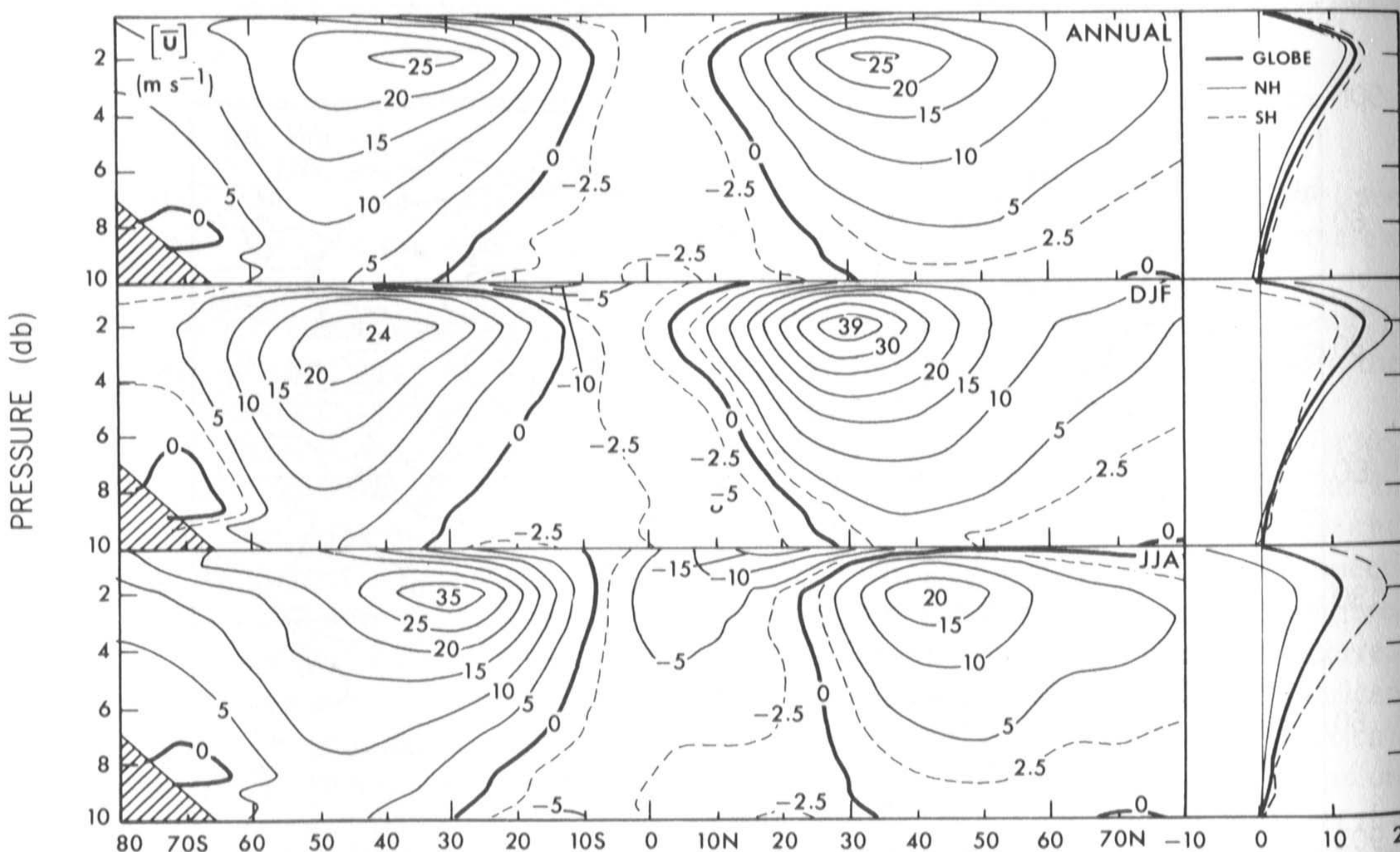


FIGURE 7.15. Zonal-mean cross sections of the zonal wind component in m s^{-1} for annual, DJF, and JJA mean conditions. Vertical profiles of the hemispheric and global mean values are shown on the right.

Peixoto and Oort, *Physics of Climate*, page 154

Atmosphere conserves…

- Energy (mechanical, heat, …)
- Mass (of air, and other gasses)
- Momentum (and angular momentum)

To describe motion, we can make use of:

- Newton's 2nd law ($\sum F = ma$)
- First Law of Thermodynamics
- We need to account for fact that Earth is rotating, spherical and there is gravity

Define some coordinates

- Position x,y,x

  But there are others!

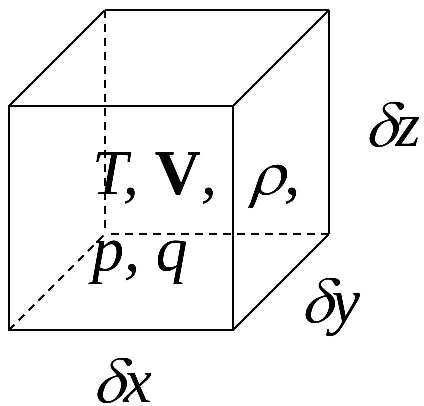
Horizontal: Longitude, latitude

Vertical: Pressure, potential temperature,….

Velocity: u, v, w
  (Change in position with time)

The atmosphere is a fluid continuum

- Can be described atmosphere by a set field variables that are continuous functions (in space and time) e.g. temperature, pressure, wind, mass, water content
- It is convenient to consider these properties applying to a "small" parcel of air



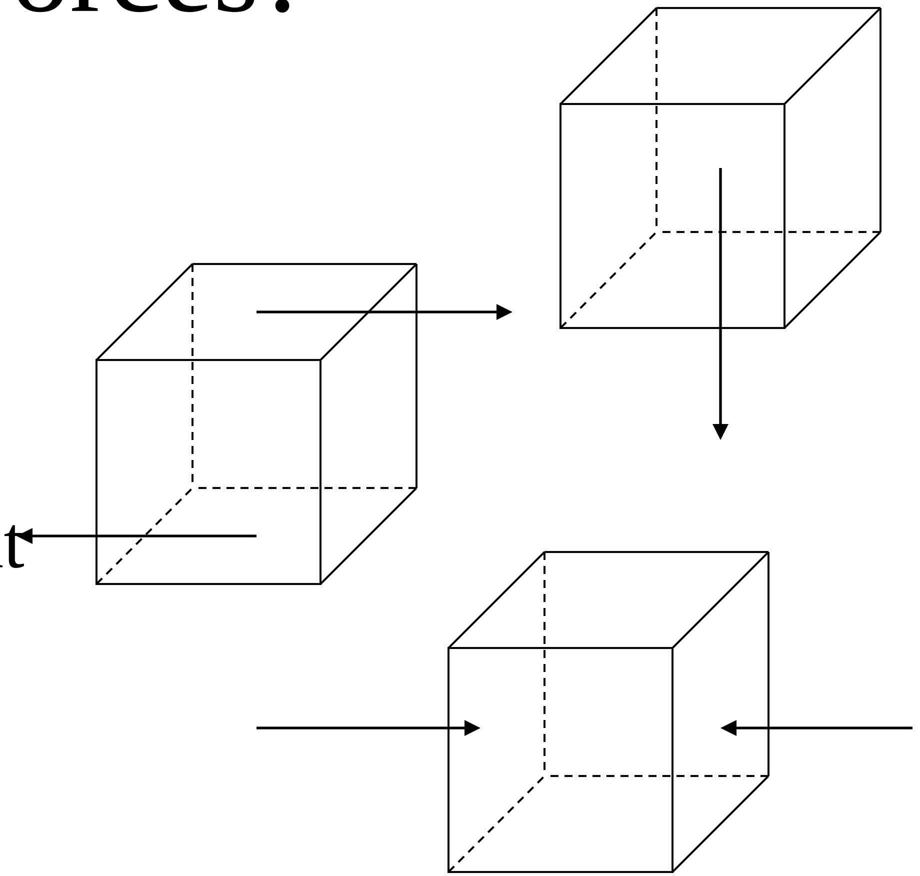
Volume $dV = dx\ dy\ dz$

Mass $dm = \rho\, dV$

•Total mass of the atmosphere $M = \int_{globe} dm$

Forces?

- F_g, Gravity
- F_v, Viscosity
- F_p, Pressure gradient



$\Sigma F = ma$

All have the same units, N

Gravity

- Newton's law of gravity

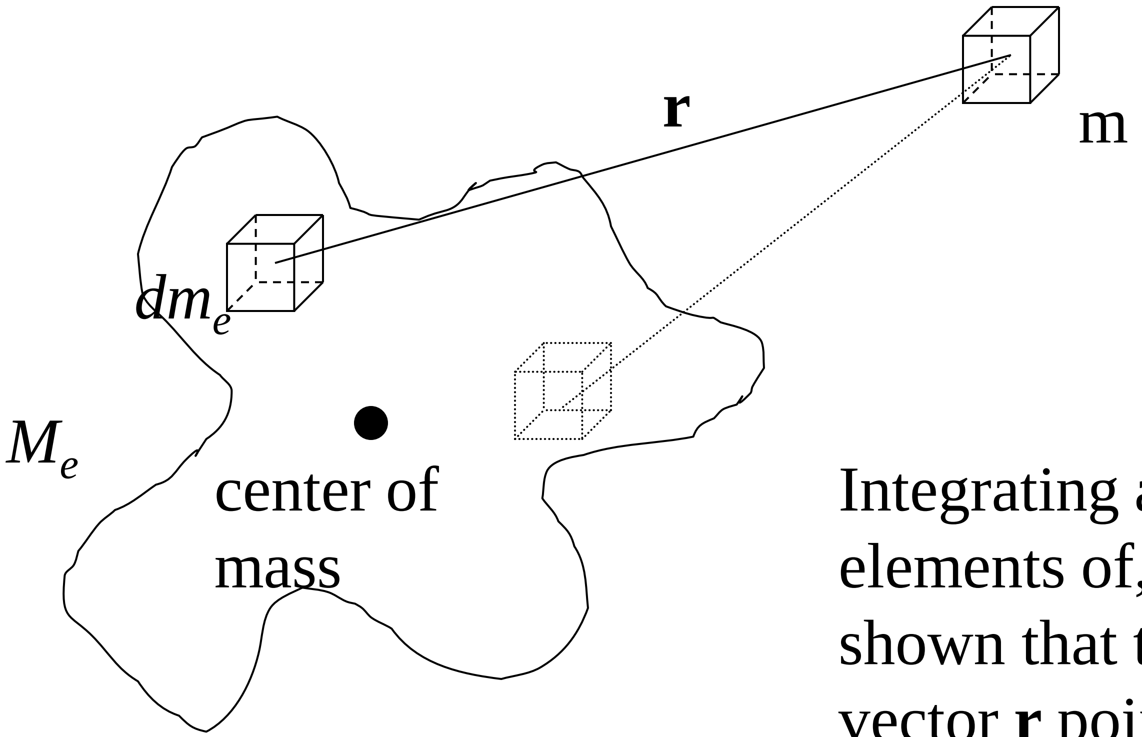
$$F = -\frac{GM_e m}{r^2}$$

M_e	mass of the Earth
m	mass of the parcel
r	Distance between center of the Earth and air parcel

So acceleration,

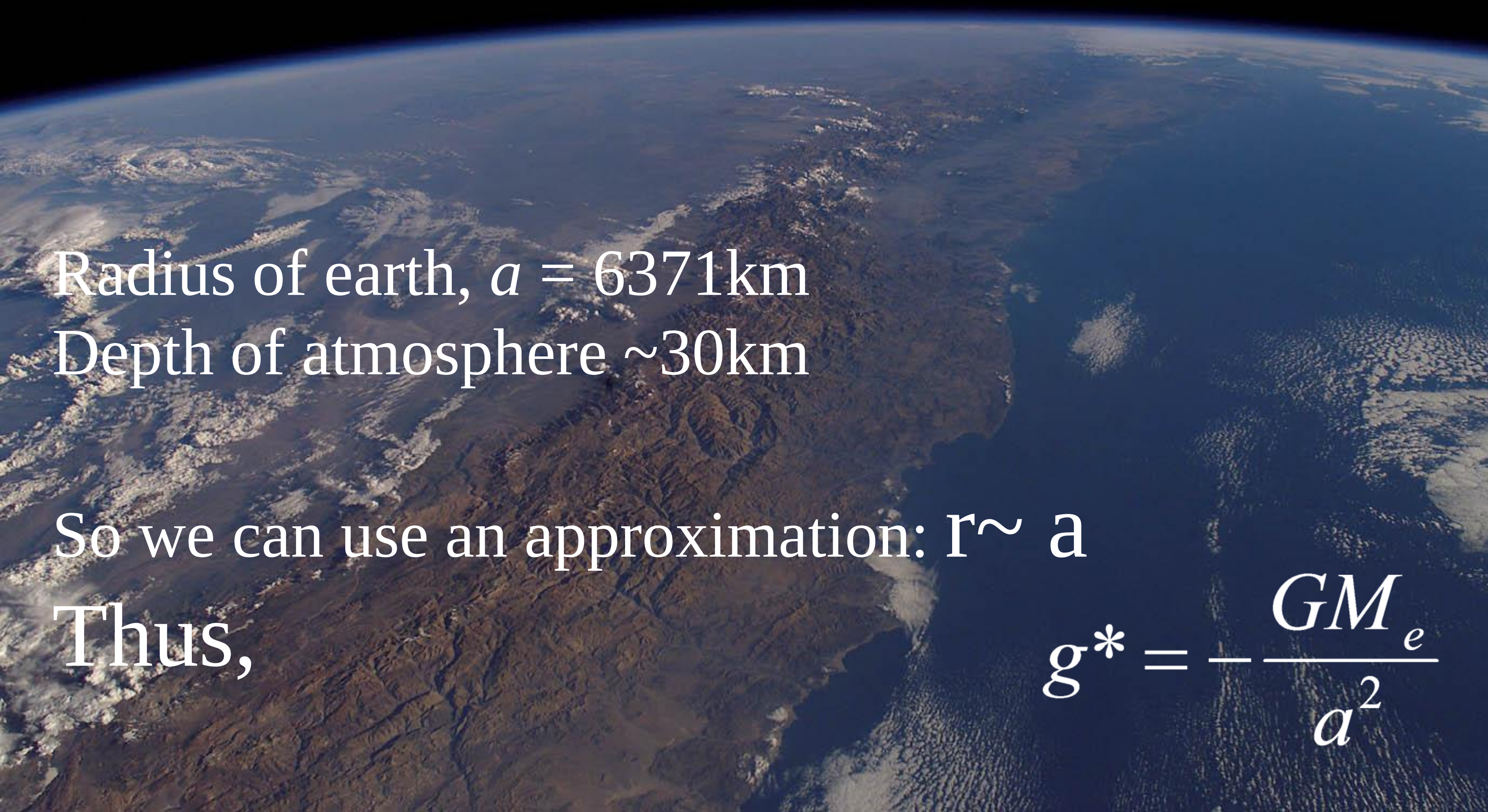
$$g_0{}^* = \frac{F}{m} = -\frac{GM_e}{r^2}$$

Directed ***exactly*** toward the Earth's center of gravity



Integrating all mass elements of, it can be shown that the direction vector **r** points to the center of mass

Earth is big, atmosphere is small



Radius of earth, a = 6371km

Depth of atmosphere ~30km

So we can use an approximation: r~ a

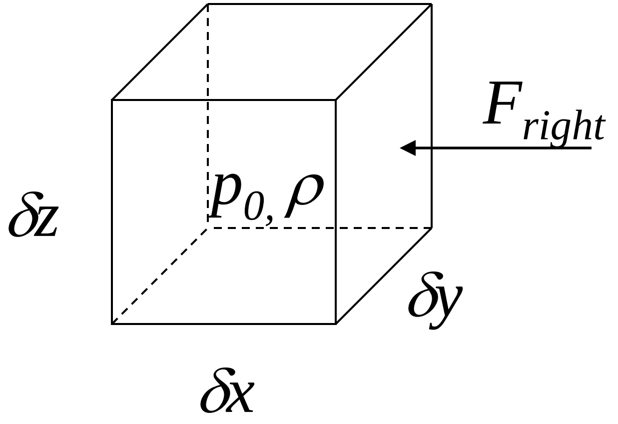
Thus,

$$g^* = -\frac{GM_e}{a^2}$$

Pressure gradient force

- Let's consider only the *x* direction
- Force exerted by air molecules colliding with "walls" of air parcel
- Taylor's expansion for pressure:

$$p = p_0 + \frac{1}{2}\frac{\partial p}{\partial x}dx + \text{higher order terms}$$



Pressure on sides:

$$p_{right} = p_0 + \frac{1}{2}\frac{\partial p}{\partial x}\delta x$$

$$p_{left} = p_0 - \frac{1}{2}\frac{\partial p}{\partial x}\delta x$$

Pressure gradient force

- Pressure is force per unit area (p = F/A)
- Force on sides: e.g., $F_{right} = -p_{right}\delta y \delta z$
- Summing left and right sides

$$F_x = F_{left} + F_{right} = \left(p_0 - \frac{1}{2}\frac{\partial p}{\partial x}\delta x \right)\delta y \delta z - \left(p_0 + \frac{1}{2}\frac{\partial p}{\partial x}\delta x \right)\delta y \delta z$$

$$F_x = -\frac{\partial p}{\partial x}\delta x \delta y \delta z$$

$$\frac{F_x}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

(recall, mass related to volume and density)

Pressure gradient vector

$$\frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \frac{F_y}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \frac{F_z}{m} = -\frac{1}{\rho}\frac{\partial p}{\partial z}$$

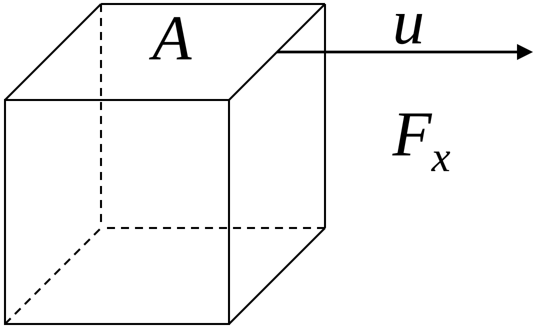
Summing components, we can write the vector form using the gradient (or “del”) operator,

$$\sum_i \frac{\mathbf{F}_i}{m} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j} + \frac{\partial p}{\partial z}\mathbf{k}\right)$$

$$\frac{\mathbf{F_p}}{m} = -\frac{1}{\rho}\nabla p$$

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Viscosity



- In a fluid with a maintained velocity profile, an acceleration (force per unit mass) is required.

- Define a shear stress τ (a force per unit area), with the dynamic viscosity μ

$$F_{xz} = \mu A \frac{\partial u}{\partial z}$$

- Using Taylor's expansion, and applying to upper and lower walls (as we did for pressure gradient)

$$\tau_{xz} = \mu \frac{\partial u}{\partial z}$$

$$\tau_{xz} = \tau_{xz0} + \frac{1}{2}\frac{\partial \tau_{xz}}{\partial z} z + \ldots$$

$$\frac{1}{\rho}\frac{\partial \tau_{xz}}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right)$$

so in vector form

$$\frac{\mathbf{F}}{m} = \frac{\mu}{\rho}\nabla^2 V$$

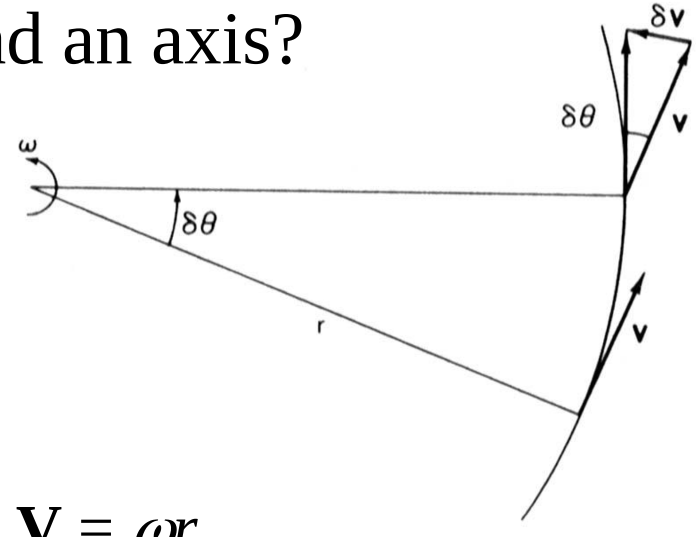
However, for most of the atmosphere, μ is small an this can be neglected – the atmosphere is almost *inviscid*. ***We will return to the sppecial case later…***

Rotating reference frames

- Newton’s Laws apply to inertial (non-accelerating) reference frames
- Rotation is non-inertial
- So we must account for rotating effects when we examine forces
- Specifically, we need to account for the fact that or coordinate system is accelerating
- This introduced “apparent” forces

Centripetal acceleration

- Acceleration is a change in velocity
- Velocity has speed and direction
- So, what is the acceleration needed for a body (e.g., an air parcel) to rotate around an axis?



Define, angular position, θ

Angular velocity, $\omega = d\theta/dt$

Simple geometry gives us,
In time δt, body rotates angle $\delta\theta$
thus we find,

$$\mathbf{V} = \omega r$$

$$\frac{d\mathbf{V}}{dt} = -\omega r^2$$

Centripetal acceleration is real
Centrifugal acceleration is not

- This is the force required to rotate a body
  (e.g. car door pushes you around the corner)
- However, if we view the air parcel from the air parcel’s point of view, we see a fictitious force – this is the centrifugal force
- Centrifugal force seems to push the parcel away from the rotation axis (*something* tries to throw you out the car door).

  [This appears as another *body force*, like gravity]

Air hockey

Summary of forces

- Gravity
- Pressure gradient force
- Friction and viscosity
  (between layers of air, or at surfaces)

Also, electrostatic forces and magnetic forces
(These both act on ionized molecules, of which there are very few below ionosphere, so we can ignore them)

- Also have Coriolis force due to rotation
  (and curvature)

Our growing big nasty equation

- The momentum equation
(conservation of momentum)

- Acceleration = pressure gradient + Coriolis
(gravity in the vertical…. And sometimes friction)

$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + fv + F_{rx}$$

$$f = 2\,\Omega \sin\phi$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - fu + F_{ry}$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Exercise 1

- 1) Solve angular momentum problem on slide 30
- 2) Holton 2.1
  A ship is steaming northward at a rate of 10 km/hr. The surface pressure increases toward the northwest at a rate of 5 Pa/km. What is the pressure tendency recorded at a nearby island station if the pressure aboard the ship decreases at a rate of 100 Pa in 3 hours?
  Hint: this problem encourages you to review partial derivatives and advection!
  To go over and hand in in the next class – THURSDAY!

Exercise 1a)

- Using conservation of angular momentum, what is the zonal (westerly) velocity of a parcel initially at rest at the equator when displaced to 30 N?

- Hint: $M = (\Omega + u/R)R^2$

$a = 6{,}391{,}000$ m, $\Omega = 2\pi$ radians per day

Influence of rotation on moving parcels

Atmosphere (air parcels) conserve angular momentum

- Consider angular momentum per unit mass (M) (which is just the angular velocity)

$$M = (\Omega + u/R)R^2$$

Ω planetary rotation rate
(2 π radians per day for Earth)

u relative (westerly) velocity

R the distance to the rotation axis
(relative angular velocity is u/R)

Earth is spherical, so R = a cos ϕ

Since Ω is constant, if we change latitude ϕ, we must also change the velocity of the parcel.
This is what we wish to quantify.

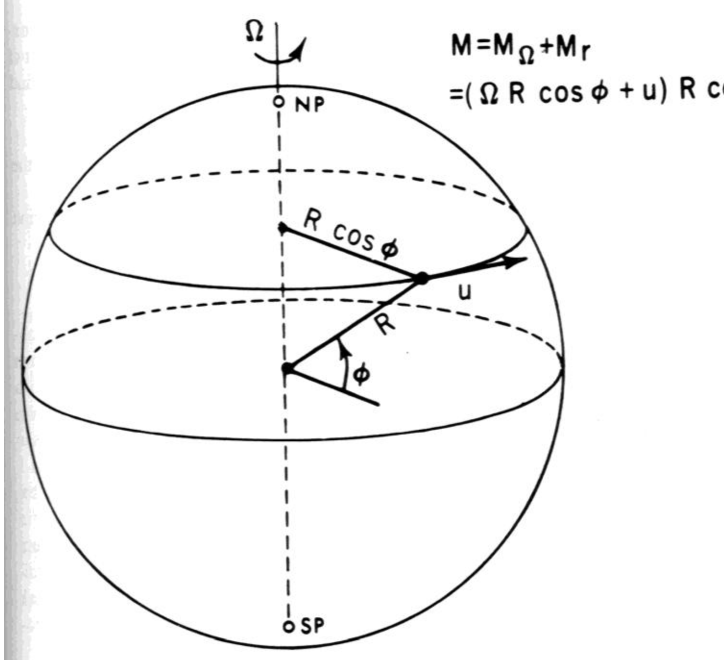


FIGURE 11.1. Schematic diagram of the angular momentum component around the ea
of rotation. NP = North Pole; SP = South Pole.